NABL 141

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GUIDELINES FOR ESTIMATION *and* EXPRESSION OF UNCERTAINTY IN MEASUREMENT

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FOREWARD

The expression of "Uncertainty in Measurements" is an integral component of the accreditation certificate being issued to the calibration laboratories. Globalization of trade and technology implies the need for interchangeability of components, which must be produced with a high degree of exactness in measurement system. This concept is equally true for all other fundamental units of measurement. The International Bureau of Weights and Measures (BIPM), in consultation with various international bodies, have arrived at a new ISO standard on Expression of Uncertainty in Measurements, in 1995.

I am glad to dedicate the document of NABL on Guidelines for Estimation and Expression of Uncertainty in Measurement to the cause of calibration laboratories in the country. I take this opportunity to congratulate the scientists who have made handsome contributions in bringing out this document based on the latest ISO standard.

New Delhi
2nd April, 2000

V. S. Ramamurthy,
Chairman, NABL
and Secretary, DST

AMENDMENT SHEET

Sl no	Page No.	Clause No.	Date of Amendment	Amendment made	Reasons	Signature QM	Signature Director
1	28	Appen-dix B	18.08.00	**"Not"** deleted from Type B evaluation in **Note**: /	Printing Mistake/ APLAC evaluation	Sd/-	Sd/-
2	29	Appen-dix B	18.08.00	Reference to GUM-additional information for high precision measurement	APLAC evaluation	Sd/-	Sd/-
3	29-30	Appen-dix B	18.08.00	Interpretation to effective degrees of freedom is added	For Better clarity on selection of effective degrees of freedom / APLAC evaluation	Sd/-	Sd/-
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1. Introduction

1.1 Purpose

The purpose of the document is to harmonize procedures for evaluating uncertainty in measurements and for stating the same in calibration certificates as are being followed by the NABL with the contemporary international approach. The document will apprise calibration laboratories of the current requirements for evaluating and reporting uncertainty and will assist accreditation bodies with a coherent assignment of test measurement capability to calibration laboratories accredited by them. The document will also provide broad guidelines to all those who are concerned with measurements about uncertainty in measurement, estimation and apportionment of uncertainty and interpretation of uncertainty. In fact, the purpose is to provide guidelines to users about contemporary requirements for global acceptance of various kinds of measurements. Attempts have been made to make the provisions of this document easy to understand and ready for implementation. The present document will replace NABL's document 141 (1992).

1.2 Scope

Provisions of this document apply to measurements of all sorts as are carried out in calibration laboratories. For specialized measurements, these may have to be supplemented by more specific details and, in some cases, appropriately modified forms of the concerned formulae. Measurements which can be treated as outputs of several correlated inputs have been excluded from the scope of this document. The document covers the following topics:

- Uncertainty – concept, sources and measures
- Definitions of related terms and phrases
- Evaluation of standard uncertainty in input estimates
- Evaluation of standard uncertainty in output estimates
- Expanded uncertainty in measurement
- Statement of uncertainty in measurement
- Apportionment of standard uncertainty
- Step by step procedure for calculating the uncertainty in measurement
- Appendix – A: Use of relevant probability distribution
- Appendix – B: Coverage factor and effective degrees of freedom
- Appendix – C: Solved examples showing the application of the method outlined here to eight specific problems in different fields.

1.3 Normative References :

This document is based primarily on the Guide to the expression of uncertainty in measurement (1993) jointly prepared by BIPM, IEC, ISO and OIML for definition of various terms and phrases. One should refer to ISO 3534-I (1993) part – I probability and general statistical terms.

1. Guidelines for estimation and statement of overall uncertainty in measurement results, NABL – 141, Department of Science and Technology, New Delhi (India), (1992).
2. Guide to the expression of uncertainty in measurement, International Bureau of Weights and Measures (BIPM), International Organization for Standardization (ISO) et. al., Switzerland, 1995.
3. International vocabulary of basic and general terms in metrology, International Bureau of Weights and Measures (BIPM), International Organization for Standardization (ISO) et. al. ., Switzerland , 1993.
4. Expression of the uncertainty of measurement in calibration, European Cooperation for Accreditation of laboratories (EAL – R-2), 1997
5. Guidelines on the evaluation and expression of the measurement uncertainty, Singapore Institute of Standards and Industrial Research, Singapore 1995.
6. International standard ISO 3534 – I, statistics – vocabulary and symbols – Part I. Probability and general statistical terms, first edition, International Organization for Standardization (ISO) , Switzerland ,1993.

2. Uncertainty – Concept, Sources and Measures

2.1 Concept

2.1.1 Quality of measurements has assumed great significance in view of the fact that measurements (in a broad sense) provide the very basis of all control actions. Incidentally, the word measurement should be understood to mean both a process and the output of that process.

2.1.2 It is widely recognized that the true value of a measurand (or a duly specified quantity to be measured) is indeterminate, except when known in terms of theory. What we obtain from the concerned measurement process is at best an estimate of or approximation to the true value. Even when appropriate corrections for known or suspected components of error have been applied, there still remains an uncertainty, that is, a doubt about how well the result of measurement represents the true value of the quantity being measured.

2.1.3 A statement of results of measurement (as a process) is complete only if it contains both the values attributed to the measurand and the uncertainty in measurement associated with that value. Without such an indication, measured results can not be compared, either among themselves or with reference values given in a specification or standard.

2.1.4 The uncertainty of measurement is a parameter, associated with the result of a measurement, that characterizes the dispersion of the true values, which could reasonably be attributed to the measurand. The parameter may be, for example, the standard deviation (or a given multiple of it), or the half-width of an interval having a stated level of confidence.

2.2 Source

2.2.1 Errors in the observed results of a measurement (process) give rise to uncertainty about the true value of the measurand as is obtained (estimated) from those results. Both systematic and random errors affecting the observed results (measurements) contribute to this uncertainty. These contributions have been sometimes referred to as systematic and random components of uncertainty respectively.

2.2.2 Random errors presumably arise from unpredictable and spatial variations of influence quantities, for example:

- the way connections are made or the measurement method employed
- uncontrolled environmental conditions or their influences
- inherent instability of the measuring equipment
- personal judgement of the observer or operator, etc.

These cannot be eliminated totally, but can be reduced by exercising appropriate controls.

2.2.3 Various other kinds of errors, recognized as systematic, are also observed. Some common type of these errors are due to:

- those reported in the calibration certificate of the reference standards /instruments used
- different influence conditions at the time of measurement compared with those prevalent at the time of calibration of the standard (quite common in length and d.c. measurements) etc.

2.2.4 It should be pointed out that errors, which can be recognized as systematic and can be isolated in one case, may simply pass of as random in another case.

2.3 Measures

2.3.1 Measurands are particular quantities subject to measurement. In calibration one usually deals with only one measurand or output quantity Y that depends upon a number of input quantities X_i (i = 1,2,…., N) according to the functional relationship,

$$Y = f(X_1, X_2, \ldots\ldots, X_N). \tag{2.1}$$

The model function f represents the procedure of the measurement and the method of evaluation. It describes how values of the output quantity Y are obtained from values of the input quantities $X_{i.}$

2.3.2 An estimate of the measurand Y (output estimate) denoted by y, is obtained from Eq. (2.1) using input estimates x_i for the values of the input quantities $X_{i,}$

$$y = f(x_1, x_2, \ldots\ldots x_N) . \tag{2.2}$$

It is understood that the input values are best estimates that have been corrected for all effects significant for the model. If not, necessary corrections have been introduced as separate input quantities.

2.3.3 The standard uncertainty of measurement associated with the output estimate y, denoted by u(y), is the standard deviation of the unknown (true) values of the measurand Y corresponding to the output estimate y. It is to be determined from the model Eq. (2.1) using estimates x_i of the input quantities X_i and their associated standard uncertainties u (x_i).

The standard uncertainty associated with estimate has the same dimension as the estimate. In some cases the relative standard uncertainty of measurement may be appropriate which is the standard uncertainty associated with an estimate divided by the modulus of that estimate and is therefore dimensionless. This concept cannot be used if the estimate equals zero.

2.3.4 The standard uncertainty of the result of a measurement, when that result is obtained from the values of a number of other quantities is termed combined standard uncertainty.

2.3.5 An expanded uncertainty is obtained by multiplying the combined standard uncertainty by a coverage factor. This, in essence, yields an interval that is likely to cover the true value of the measurand with a stated high level of confidence.

3. Definitions of related terms and phrases

The guide explains explicitly a large number of metrological terms which are used in practice. A few terms of general interest have been taken from the “International Vocabulary of Basic and General terms in Metrology” and EAL document [3-4]. To facilitate the reader, various terms and phrases are arranged in alphabetical order

accepted reference value

a value that serves as an agreed upon reference for comparison.

accuracy of measurement

the closeness of agreement between a test result and the accepted reference value

arithmetic mean

The sum of values divided by the number of values

combined standard uncertainty (u_c)

standard uncertainty of the result of a measurement when that result is obtained from the values of a number of other quantities, equal to the positive square root of a sum of terms, the terms being the variances or covariances of these other quantities weighted according to how the measurement result varies with changes in these quantities

conventional true value (of a quantity)

a value of a quantity which for a given purpose, may be substituted for the true value

correction

value added algebraically to the uncorrected result of a measurement to compensate for systematic error

correction factor

numerical factor by which the uncorrected result of a measurement is multiplied to compensate for a systematic error

correlation

the relationship between two or several random variables within a distribution of two or more random variables

correlation coefficient

the ratio of the covariance of two random variables to the product of their standard deviations.

covariance

The sum of the products of the deviations of x_i and y_i from their respective averages divided by one less than the number of observed pairs:

$$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) \qquad (3.1)$$

Where n is number of observed pairs

coverage factor (k)

numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty

coverage probability or confidence level

the value of the probability associated with a confidence interval or a statistical coverage interval

degrees of freedom (ν)

the number of terms in a sum minus the number of constraints on the terms of the sum

errors

In general, a measurement has imperfections that give rise to an error in the measurement result. An error is viewed as having two components, namely, a random component and systematic component.

error in measurement

result of a measurement minus accepted reference value (of the characteristic)

estimation

the operation of assigning, from the observations in a sample, numerical values to the parameters of a distribution chosen as the statistical model of the population from which this sample is taken

estimate

the value of a statistic used to estimate a population parameter

expanded uncertainty (U)

quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand

expectation

the expectation of a function g(z) over a probability density function p(z) of the random variables z is defined by

$$E[g(z)] = \int g(z)p(z)dz \tag{3.2}$$

the expectation of the random variable z, denoted by μ_z and which is also termed as the expected value or the mean of z. It is estimated statistically by $\bar{z}$, the arithmetic mean or average of n independent observations z_i of the random variable z, the probability density function of which is p(z)

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i \tag{3.3}$$

experimental standard deviation [s(q_j)]

for a series of n measurements of the same measurand, the quantity s(q_j) characterizing the dispersion of the results and given by the formula :

$$s(q_j) = \sqrt{\frac{\sum_{j=1}^{n}\left(q_j - \bar{q}\right)^2}{n-1}} \tag{3.4}$$

q_j being the result of the jth measurement and $\bar{q}$ being the arithmetic mean of the n results considered.

measurand

a quantity subject to measurement

probability distribution

a function giving the probability that a random variable takes any given value or belongs to a given set of values

probability density function

the derivative (when it exits) of the distribution function :

$$f(x) = dF(x)/dx \tag{3.5}$$

f(x)dx is the probability element

$$f(x)\,dx = Pr(x < X < x + dx) \tag{3.6}$$

probability function

a function giving for every value x, the probability that the random variable X takes value x :

$$F(x) = Pr(X = x) \tag{3.7}$$

random error

result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions

Notes :

1. random error is equal to error minus systematic error
2. because only a finite number of measurements can be made,

it is possible to determine only one estimate of random error

random variable

a variable that may take any of the values of a specified set of values and with which is associated a probability distribution

repeatability (of results of measurements)

closeness of the agreement between the results of successive measurements of the same measurand carried out under repeatability conditions

repeatability conditions

conditions where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short interval of time.

reproducibility (of results of measurements)

closeness of the agreement between the results of the measurements of the same measurand carried out under reproducibility conditions .

reproducibility conditions

conditions where test results are obtained with the same method on identical test items in different laboratories with different operators using different equipment

results of a measurement

value attributed to a measurand, obtained by measurement

Note:

Complete statement of the result of a measurement includes information about uncertainty in measurement

sensitivity coefficient associated with an input estimate (c_i)

the differential change in the output estimate generated by the differential change in that input estimate

standard deviation (σ)

the positive square root of the variance

standard uncertainty

uncertainty of the result of a measurement expressed as a standard deviation

systematic error

mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus acceptance reference value of the measurand.

Note:
Systematic error is equal to error minus random error

true value (of a quantity)

the value which characterized a quantity perfectly defined in the conditions which exist when that quantity is considered

Note:

The true value is a theoretical concept, and, in general, can not be known exactly

Type A evaluation (of uncertainty)

Method of evaluation of uncertainty by the statistical analysis of series of observations

Type B evaluation (of uncertainty)

Method of evaluation of uncertainty by means other than the statistical analysis of series of observations.

uncertainty (in measurement)

parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

variance

A measure of dispersion, which is the sum of the squared deviations of observations from their average divided by one less than the number of observations.

4. Evaluation of standard uncertainty in Input estimates

4.1 General considerations

4.1.1 The uncertainty of measurement associated with the input estimates is evaluated according to either a "Type A" or a "Type B" method of evaluation. The Type A evaluation of standard uncertainty is the method of evaluating the uncertainty by the statistical analysis of a series of observations. In this case the standard uncertainty is the experimental standard deviation of the mean that follows from an averaging procedure or an appropriate regression analysis. The Type B evaluation of standard uncertainty is the method of evaluating the uncertainty by means other than the statistical analysis of a series of observations. In this case the evaluation of the standard uncertainty is based on some other scientific knowledge.

Examples:

Case – I : Digital multimeter (DMM)

Let us consider, an experiment in which a high accuracy reference standard e.g. a 6 ½ digit stable meter calibrator is used to calibrate a device of much lower accuracy like 4 ½ digit DMM . The readings of the test DMM may remain unchanged or undergo flicker ±1 count due to its digitizing process. In this case, the Type A evaluation of the uncertainty may be taken to be negligible, and the uncertainty on account of repeatable observations can be treated as Type B on the basis of the resolution error of the test DMM.

Case – II : Length Bar

While calibrating a length bar by comparison method, one has to include the component of uncertainty associated with the thermal expansion coefficient $\left[\alpha = \delta\ell/\ell\right]$ in the uncertainty budget. Usually, α for the test and standard is taken from handbook or as per manufacturers specification, in this case, although the estimation of uncertainty in temperature measurement is Type A but the estimation of uncertainty in α is Type B. However, in a special case where high precision is needed, in situ measurement of thermal expansion is carried out. In such a case, the evaluation of uncertainty in both temperature and α are of Type A.

4.2 Type A evaluation of standard uncertainty

4.2.1 Type A evaluation of standard uncertainty applies to situation when several independent observations have been made for any of the input quantities under the same conditions of measurement. If there is sufficient resolution in the measurement process, there will be an observable scatter or spread in the values obtained.

4.2.2 Let us denote by Q the repeatedly measured input quantity X_i. With n statistically independent observations (n > 1), the estimate of Q is $\bar{q}$, the arithmetic mean of the individual observed values q_j (j = 1, 2,…….n).

$$\bar{q} = \frac{1}{n}\sum_{j=1}^{n} q_j \qquad (4.1)$$

The uncertainty of measurement associated with the estimate $\bar{q}$ is evaluated according to one of the following methods

4.2.3 An estimate of the variance of the underlying probability distribution of q is the experimental variance s^2 (q) of values q_j given by,

$$s^2(q) = \frac{1}{n-1}\sum_{j=1}^{n}\left(q_j - \bar{q}\right)^2 \qquad (4.2)$$

The positive square root of s^2 (q) is termed experimental standard deviation. The best estimate of the variance of the arithmetic mean $\bar{q}$ is given by

$$s^2\left(\bar{q}\right) = \frac{s^2(q)}{n} \qquad (4.3)$$

Table 4.1: Data for calculation of mean and standard deviation of temperature:

Observation numbers	**Temperature ^{0}C**	$\left(t_j - \bar{t}\right)\times 10^{-2}$ ^0C	$\left(t_j - \bar{t}\right)^2 \times 10^{-4}$ $\left(^0C\right)^2$
1	90.68	-4	16
2	90.83	11	121
3	90.79	7	49
4	90.64	-8	64
5	90.63	-9	81
6	90.94	22	484
7	90.60	-12	144
8	90.68	-4	16
9	90.76	4	16
10	90.65	-7	49
Total	**907.2**	**0**	**1040**

The positive square root of s^2 ($\bar{q}$) is termed as estimated standard error of the mean. The standard uncertainty u ($\bar{q}$) associated with the input estimate $\bar{q}$ is the standard error.

$$u\left(\bar{q}\right) = s\left(\bar{q}\right) \qquad (4.4)$$

4.2.4 For a measurement that is well-characterized and under statistical control a combined or pooled estimate of variance s^2_p may be available from several sets of repeat measurements that characterizes the dispersion better than the estimated variance obtained from a single set of observations. If in such a case the value of the input quantity Q is determined as the arithmetic mean $\bar{q}$ of n independent observations, the variance of the mean may be estimated by

$$s^2(\bar{q}) = \frac{s_p^2}{n} \tag{4.5}$$

The standard uncertainty is deduced from the value given by Eq. (4.4)

Example:

Table (4.1) is shown the data from a temperature measurement. We now estimate different parameters as follows:

Mean Temperature:

$$\bar{t} = \frac{\left(\sum_{j=1}^{n} t_j\right)}{n} = 90.72 \ \ ^\circ C \tag{4.6}$$

The best estimate of temperature is therefore:

$$\bar{t} = 90.72 \ \ ^\circ C \tag{4.7}$$

Standard Deviation:

$$s(t) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(t_j - \bar{t})^2} = \sqrt{\frac{1}{9}(1040 \times 10^{-4})} = 10.75 \times 10^{-2} \ \ ^\circ C \tag{4.8}$$

Standard error of the mean:

$$s(\bar{t}) = \sqrt{\frac{s^2(t)}{n}} = \frac{10.75 \times 10^{-2}}{\sqrt{10}} = 3.40 \times 10^{-2} \ ^\circ C \tag{4.9}$$

Standard uncertainty:

$$u(\bar{t}) = 3.40 \times 10^{-2} \ \ ^\circ C \tag{4.10}$$

Degrees of freedom (ν)

$$\nu = n - 1 = 10 - 1 = 9 \tag{4.11}$$

4.3 Type B evaluation of standard uncertainty

4.3.1 The Type B evaluation of standard uncertainty is the evaluation of the uncertainty associated with an estimate x_i of an input quantity X_i by means other than the statistical analysis of a series of observations. The standard uncertainty $u(x_i)$ is evaluated by scientific judgment based on all available information on the possible variability of X_i.

Values belonging to this category may be derived from

- previous measurement data ;
- experience with or general knowledge of the behaviour and properties of relevant materials and instruments ;
- manufacturer's specifications ;
- data provided in calibration and other certificates;
- uncertainties assigned to reference data taken from handbooks.

4.3.2 The proper use of the available information for a Type B evaluation of standard uncertainty of measurement calls for insight based on experience and general knowledge. It is a skill that can be learned with practice. A well-based Type B evaluation of standard uncertainty can be as reliable as a Type A evaluation of standard uncertainty, especially in a measurement situation where a Type A evaluation is based only on a comparatively small number of statistically independent observations. The following cases must be discerned:

(a) When only a single value is known for the quantity X_i, e.g. a single measured value, a resultant value of a previous measurement, a reference value from the literature, or a correction value, this value will be used for x_i. The standard uncertainty $u(x_i)$ associated with x_i is to be adopted where it is given. Otherwise it has to be calculated from unequivocal uncertainty data. If data of this kind are not available, the uncertainty has to be evaluated on the basis of experience taken as it may have been stated (often in terms of an interval corresponding to expanded uncertainty).

(b) When a probability distribution [see Appendix – A] can be assumed for the quantity X_i, based on theory or experience, then the appropriate expectation or expected value and the standard deviation (σ) of this distribution have to be taken as the estimate x_i and the associated standard uncertainty $u(x_i)$, respectively.

Examples:

In cases, where the uncertainty is quoted to be particular multiple of standard deviation (σ), the multiple becomes the specific factor (see Appendix – A).

Case I:

A calibration certificate states that the mass of a given body of 10 kg is 10.000650 kg. The uncertainty at 2 σ (at confidence level of 95 .45 %) is given by 300 mg. In such a case, the standard uncertainty is simply,

$$u(m) = 300 / 2 = 150 \text{ mg} \quad (4.12)$$

and estimated variance is

$$u^2(m) = 0.0225 \text{ g}^2 \quad (4.13)$$

Case II:

Suppose in the above example, the quoted uncertainty defines an interval having a 90% level of confidence. The standard uncertainty is then

$$u(m) = 300 / 1.64 = 182.9 \text{ mg} \quad (4.14)$$

Where we have taken 1.64 as the factor corresponding to the above level of confidence, assuming the normal distribution unless otherwise stated.

Case III:

A calibration certificate states that the resistance of a standard resistor, R_s of nominal value 10 Ω is 10.000742 Ω $\pm$ 129 $\mu\Omega$ at 23 ^{0}C and that the quoted uncertainty of 129 $\mu\Omega$ defines an interval having a level of confidence of 99%. The standard uncertainty of the resistor may be taken as

$$u(R_s) = 129 \ \mu\Omega / 2.58 = 50 \ \mu\Omega \quad (4.15)$$

Therefore, in this case, specific factor is 2.58. The corresponding relative standard uncertainty

$$u(R_s)/ R_s = 5 \times 10^{-6} \quad (4.16)$$

The estimated variance is

$$u^2 = (50 \ \mu\Omega)^2 = 2.5 \times 10^{-9} \ \Omega^2 \quad (4.17)$$

Case IV:

A calibration certificate states that the length of a standard slip gauge (SG) of nominal value 50 mm is 50.000002 mm. The uncertainty of this value is 72 nm, at confidence level of 99.7 % (corresponding to 3 times of standard deviation). The standard uncertainty of the standard slip gauge is then given by

$$u(SG) = 72 \text{ nm} /3 = 24 \text{ nm} \quad (4.18)$$

(c) If only upper and lower limits a_+ and a_- can be estimated for the value of the quantity X_i (e.g. manufacturer's specifications of a measuring instrument, a temperature range, a rounding or truncation error resulting from automated data reduction), a probability distribution with constant probability density between these limits (rectangular probability distribution) has to be assumed for the possible variability of the input quantity X_i . According to case (b) above this leads to

$$x_i = \frac{1}{2}\left(a_+ + a_-\right) \tag{4.19}$$

for the estimated value and

$$u^2(x_i) = \frac{1}{12}\left(a_+ - a_-\right)^2 \tag{4.20}$$

for the square of the standard uncertainty . If the difference between the limiting values is denoted by $2a$, Eq. (4.20) yields

$$u^2(x_i) = \frac{1}{3}(a)^2 \tag{4.21}$$

Examples:

The specifications of a dial type pressure gauge are as follows :

Range : 0 to 10 bar,
Scale : 1 division = 0.05 bar,
Resolution : ½ division = 0.025 bar
Accuracy : ± 0.25 % Full Scale Deflection

Assuming that with the above specifications, there is an equal probability of the true value lying anywhere between the upper (a_+) and lower (a_-) limits. Therefore, for rectangular distribution,

$$a = \frac{(a_+ - a_-)}{2} \tag{4.22}$$

Here ,

$$a_+ = (0.25\% \times 10)\, bar = 0.025\, bar \tag{4.23}$$

and

$$a_- = -(0.25\% \times 10)\, bar = -0.025\, bar \tag{4.24}$$

$$a = 0.05/2 = 0.025\, bar. \tag{4.25}$$

Hence the standard uncertainty is given by ,

$$u = \frac{a}{\sqrt{3}} = \frac{0.025}{\sqrt{3}} = 0.0144\, bar \tag{4.26}$$

5. Evaluation of standard uncertainty in output estimate

5.1 For uncorrelated input quantities the square of the standard uncertainty associated with the output estimate y is given by,

$$u^2(y) = \sum_{i=1}^{n} u_i^2(y) \qquad (5.1)$$

The quantity $u_i(y)$ (i = 1, 2,…, n) is the contribution to the standard uncertainty associated with the output estimate y resulting from the standard uncertainty associated with the input estimate x_i,

$$u_i(y) = c_i\, u(x_i) \qquad (5.2)$$

where c_i is defined as sensitivity coefficients associated with the input estimate x_i i.e. the partial derivative of the model function f with respect to X_i , evaluated at the input estimates x_i .

$$c_i = (\partial f / \partial x_i) = (\partial f / \partial X_i) \text{ at } X_i = x_i \qquad (5.3)$$

5.2 The sensitivity coefficient c_i, describes the extent to which the output estimate y is influenced by variations of the input estimate x_i. It can be evaluated from the function f by Eq. (5.3) or by using numerical methods, i.e. by calculating the change in the output estimate y due to a change in the input estimate x_i of + $u(x_i)$ and -$u(x_i)$ and taking as the value of c_i the resulting difference in y divided by $2u(x_i)$. Sometimes it may be more appropriate to find the change in the output estimate y from an experiment by repeating the measurement at e.g. $x_i \pm u(x_i)$.

5.3 If the model functions is a sum or difference of the input quantities X_i,

$$f(X_1, X_2, \ldots\ldots X_N) = \sum_{i=1}^{N} p_i X_i \qquad (5.4)$$

the output estimate according to Eq. (2.2) is given by the corresponding sum or difference of the input estimate

$$y = \sum_{i=1}^{N} p_i x_i \qquad (5.5)$$

whereas the sensitivity coefficients equal to p_i and Eq. (5.1) converts to

$$u^2(y) = \sum_{i=1}^{N} p_i^2 u^2(x_i) \qquad (5.6)$$

5.4 If the model function f is a product or quotient of the input quantities X_I

$$f(X_1, X_2, \ldots\ldots, X_N) = c\prod_{i=1}^{N} X_i^{p_i} \tag{5.7}$$

the output estimate again is the corresponding product or quotient of the input estimates

$$y = c\prod_{i=1}^{N} X_i^{p_i} \tag{5.8}$$

The sensitivity coefficients equal $p_i y/x_i$ in this case and an expression analogous to Eq. (5.6) is obtained from Eq. (5.1), if relative standard uncertainties w(y) = u(y)/y and w (x_i) = u (x_i) / x_i are used,

$$w^2(y) = \sum_{i=1}^{n} p_i^2 w^2(x_i) \tag{5.9}$$

6. Expanded uncertainty in measurement

6.1 Calibration laboratories shall state an expanded uncertainty in measurement (U), obtained by multiplying the standard uncertainty u(y) of the output estimate y by a coverage factor k,

$$U = ku\,(y) \tag{6.1}$$

In cases where a normal (Gaussian) distribution can be attributed to the measurand and the standard uncertainty associated with the output estimate has sufficient reliability, the standard coverage factor k = 2 shall be used. The assigned expanded uncertainty corresponds to a coverage probability of approximately 95 %.

6.2 The assumption of a normal distribution cannot always be easily confirmed experimentally. However, in the cases where several (i.e. $N \geq 3$) uncertainty components derived from well–behaved probability distributions of independent quantities, e.g. normal distributions or rectangular distributions, contribute to the standard uncertainty associated with the output estimate by comparable amounts, the conditions of the central limit theorem are met and it can be assumed to a high degree of approximation that the distribution of the output quantity is normal.

6.3 The reliability of the standard uncertainty assigned to the output estimate is determined by its effective degrees of freedom (see Appendix B). However, the reliability criterion is always met if none of the uncertainty contributions is obtained from a Type A evaluation based on less than ten repeated observations.

6.4 If one of these conditions (normality or sufficient reliability) is not fulfilled, the standard coverage factor k = 2 can yield an expanded uncertainty corresponding to a coverage probability of less than 95 %. In these cases, in order to ensure that a value of the expanded uncertainty is quoted corresponding to the same coverage probability as in the normal case, other procedures have to be followed. The use of approximately the same coverage probability is essential whenever two results of measurement of the same quantity have to be compared, e.g. when evaluating the results of an interlaboratory comparison or assessing compliance with a specification.

6.5 Even if a normal distribution can be assumed, it may still occur that the standard uncertainty associated with the output estimate is of insufficient reliability. If, in this case, it is not expedient to increase the number n of repeated measurements or to use a Type B evaluation instead of the Type A evaluation of poor reliability, the method given in Appendix – B should be used.

6.6 For the remaining cases, i.e. all cases where the assumption of a normal distribution cannot be justified, information on the actual probability distribution of the output estimate must be used to obtain a value of the coverage factor k that corresponds to a coverage probability of approximately 95 %.

7. Statement of uncertainty in measurement

7.1 In calibration certificates the complete result of the measurement consisting of the estimate y of the measurand and the associated expanded uncertainty U shall be given in the form (y ±U). To this an explanatory note must be added which in the general case should have the following content: **The reported expanded uncertainty in measurement is stated as the standard uncertainty in measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.**

7.2 However, in cases where the procedure of Appendix A has been followed, the additional note should read as follows: **The reported expanded uncertainty in measurement is stated as the standard uncertainty in measurement multiplied by the coverage factor k which for a t-distribution with ν_{eff} effective degrees of freedom corresponds to a coverage probability of approximately 95 %. (See Appendix – B).**

7.3 The numerical value of the uncertainty in measurement should be given to at most two significant figures. The numerical value of the measurement result should in the final statement normally be rounded to the least significant figure in the value of the expanded uncertainty assigned to the measurement result. For the process of rounding, the usual rules for rounding of numbers have to be used. However, if the rounding brings the numerical value of the uncertainty in measurement down by more than 5 %, the rounded up value should be used.

8. Apportionment of standard uncertainty

8.1 The uncertainty analysis for a measurement-sometimes called the Uncertainty Budget of the measurement-should include a list of all sources of uncertainty together with the associated standard uncertainties of measurement and the methods of evaluating them. For repeated measurements the number n of observations also has to be stated. For the sake of clarity, it is recommended to present the data relevant to this analysis in the form of a table. In this table all quantities should be referenced by a physical symbol X_i, or a short identifier. For each of them at least the estimate x_i, the associated standard uncertainty in measurement u (x_i), the sensitivity coefficient c_i and the different uncertainty contributions $u_i(y)$ should be specified. The degrees of freedom have to be mentioned. The dimension of each of the quantities should also be stated with the numerical values in the table.

8.2 A formal example of such an arrangement is given as Table (8.1) applicable for the case of uncorrelated input quantities. The standard uncertainty associated with the measurement result u(y) given in the bottom right corner of the table is the root sum square of all the uncertainty contributions in the outer right column. Similarly, ν_{eff} has to be evaluated as mentioned in Appendix –B.

Table 8.1: Schematic view of an Uncertainty Budget

Source of Uncertainty X_i	**Estimates x_i**	**Limits $\pm \Delta x_i$**	**Probability Distribution - Type A or B**	**Standard Uncertainty $u(x_i)$**	**Sensitivity coefficient c_i**	**Uncertainty contribution $u_i(y)$**	**Degree of freedom ν_i**
X_1	x_1	Δx_1	-Type A or B	$u(x_1)$	c_1	$u_1(y)$	ν_1
X_2	x_2	Δx_2	-Type A or B	$u(x_2)$	c_2	$u_2(y)$	ν_2
X_3	x_3	Δx_3	-Type A or B	$u(x_3)$	c_3	$u_3(y)$	ν_3
X_N	x_N	Δx_N	-Type A or B	$u(x_N)$	c_N	$u_N(y)$	ν_N
Y	y					$u_c(y)$	ν_{eff}

9. Step-by-step procedure for calculating the uncertainty in measurement

The following is a guide to the use of this document in practice:

Step 1 Express in mathematical terms the dependence of the measured (output quantity) Y on the input quantities X_i according to Eq. (2.1). In the case of a direct comparison of two standards the equation may be very simple, e.g.

$$Y = X_1 + X_2 \qquad (9.1)$$

Step 2 Identify and apply all significant corrections to the input quantities.

Step 3 List all sources of uncertainty in the form of an uncertainty analysis in accordance with Section 8.

Step 4 Calculate the standard uncertainty $u(\overline{q})$ for repeatedly measured quantities in accordance with sub-section 4.2.

Step 5 For single values, e.g. resultant values of previous measurements, correction values or values from the literature, adopt the standard uncertainty where it is given or can be calculated according to paragraph 4.3.2(a). Pay attention to the uncertainty representation used. If no data are available from which the standard uncertainty can be derived, state a value of u (x_i) on the basis of scientific experience.

Step 6 For input quantities for which the probability distribution is known or can be assumed, calculate the expectation and the standard uncertainty u (x_i) according to paragraph 4.3.2 (b). If only upper and lower limits are given or can be estimated, calculate the standard uncertainty u (x_i) in accordance with paragraph 4.3.2(c).

Step 7 Calculate for each input quantity X_i the contribution u_i (y) to the uncertainty associated with the output estimate resulting from the input estimate x_i according to Eqs. (5.2) and (5.3) and sum their squares as described in Eq. (5.1) to obtain the square of the standard uncertainty u(y) of the measurand.

Step 8 Calculate the expanded uncertainty U by multiplying the standard uncertainty u(y) associated with output estimate by a coverage factor k chosen in accordance with Section 6.

Step 9 Report the result of the measurement comprising the estimate y of the measurand, the associated expanded uncertainty U and the coverage factor k in the calibration certificate in accordance with Section 7.

Appendix A

Probability distribution

A.1 Normal distribution

The probability density function p(x) of the normal distribution is as follows:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-(x-\mu)^2 / 2\sigma^2\right], \qquad -\infty < x < +\infty \qquad (A.1)$$

Where μ is the mean and σ is the standard deviation. Figure (A.1) represents such a distribution.

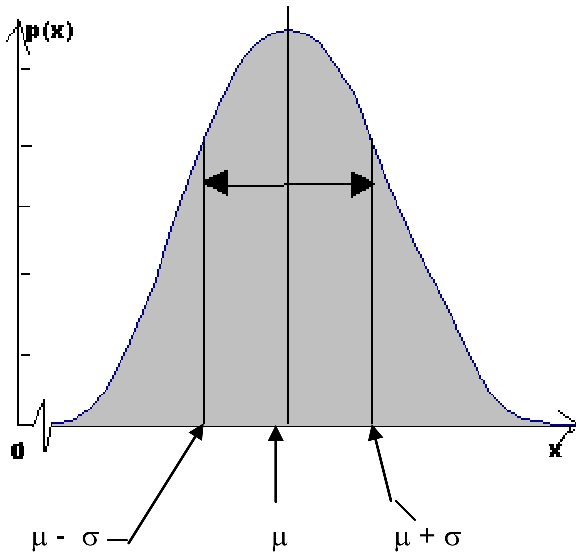

Figure A.1: Schematic view of the normal (Gaussian) distribution

A.1.1 When to use normal distribution

In some situation, the quoted uncertainty in an input or output quantity is stated along with level of confidence. In such cases, one has to find the value of coverage factor so that the quoted uncertainty may be divided by this coverage factor to obtain the value of standard uncertainty. The value of the coverage factor depends upon the distribution of the (input or output) quantity. In the absence of any specific knowledge about this distribution, one may assume it to be normal. Values of the coverage factor for various level of confidence for a normal distribution are as follows:

Table A.1: Confidence Level and the corresponding Coverage factor (k)

Confidence level	68.27 %	90 %	95 %	95.45 %	99 %	99.73 %
Coverage factor (k)	1.000	1.645	1.960	2.000	2.576	3.000

If based on available information, it can be stated that there is 50 % chance that the value of input quantity X_i lies in the interval between a_- and a_+ and also it is assumed that the distribution of X_i is normal, then the best estimate of X_i is:

$$x_i = a = (a_- + a_+)/2 \text{, with } u(x_i) = 1.48\, a \quad \text{(A.2)}$$

If based on available information, it can be stated that there is 68% chance that the value of input quantity X_i lies in the interval of a_- and a_+; and also it is assumed that the distribution of X_i is normal, then the best estimate of X_i is :

$$x_i = a, \text{ with } u(x_i) = a \quad \text{(A.3)}$$

A.2 Rectangular distribution

The probability density function p(x) of rectangular distribution is as follows:

$$P(x) = \frac{1}{2a}, a_- < x < a_+ \text{, } where\ a = (a_+ - a_-)/2 \quad \text{(A.4)}$$

Figure (A.2) represents such a distribution. The expectation of X_i is given as x_i

$$E(X_i) = x_i = (a_+ + a_-)/2 \quad \text{(A.5)}$$

and its variance is

$$Var(X_i) = a^2/3 \text{, } where\ a = (a_+ - a_-)/2 \quad \text{(A.6)}$$

A.2.1 When to use rectangular distribution

In cases, where it is possible to estimate only the upper and lower limits of an input quantity X_i and there is no specific knowledge about the concentration of values of X_i within the interval , one can only assume that it is equally probable for X_i to lie anywhere within this interval . In such a situation rectangular distributions is used.

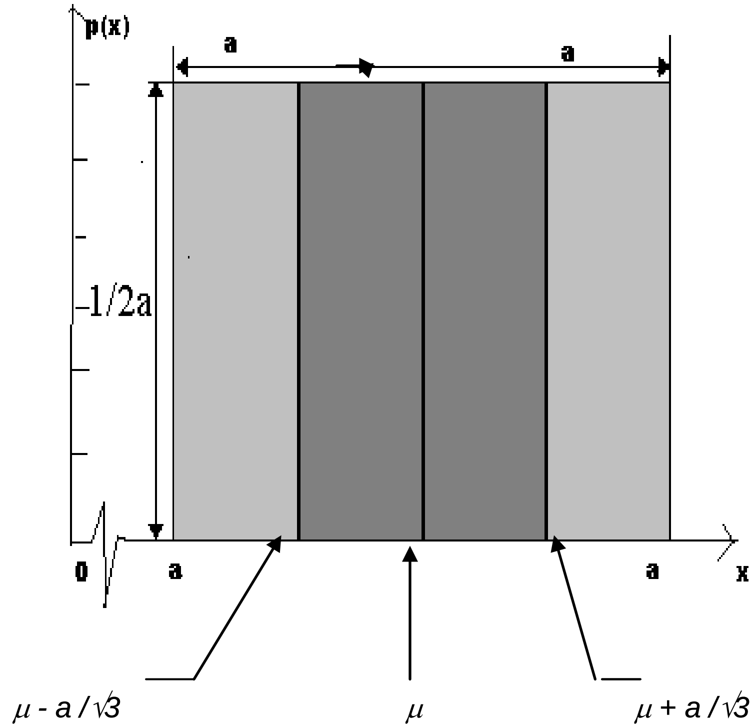

Figure A.2: Schematic view of the rectangular distribution

A.3 Symmetrical Trapezoidal Distribution

The above rectangular distribution assumes that X_i can assume any value within the interval with the same probability. However, in many realistic cases, it is more reasonable to assume that X_i can lie anywhere within a narrower interval around the midpoint with the same probability while values nearer the bounds are less and less likely to occur. For such cases, the probability distribution is represented by a symmetric trapezoidal distribution function having equal sloping sides (an isosceles trapezoid), a base of width $a_+ - a_- = 2a$, and a top of width $2\beta a$, where $0 \leq \beta \leq 1$ is used .

The expectation of X_i is given as:

$$E(X_i) = (a_+ + a_-)/2 \,, \tag{A.7}$$

and its variance is

$$Var(X_i) = a^2(1+\beta)^2/6 \tag{A.8}$$

When $\beta \to 1$, the symmetric trapezoidal distribution is reduced to a rectangular distribution.

A.3.1 Triangular Distribution

When β = 0, the symmetric trapezoidal distribution is reduced to a triangular distribution. Figure (A.3) shows such a distribution. When the greatest concentration of the values is at the center of the distribution, then one must use the triangular distribution.

The expectation of X_i is given as,

$$E(X_i) = (a_+ + a_-)/2\,, \tag{A.9}$$

and its variance is

$$Var(X_i) = a^2/6 \tag{A.10}$$

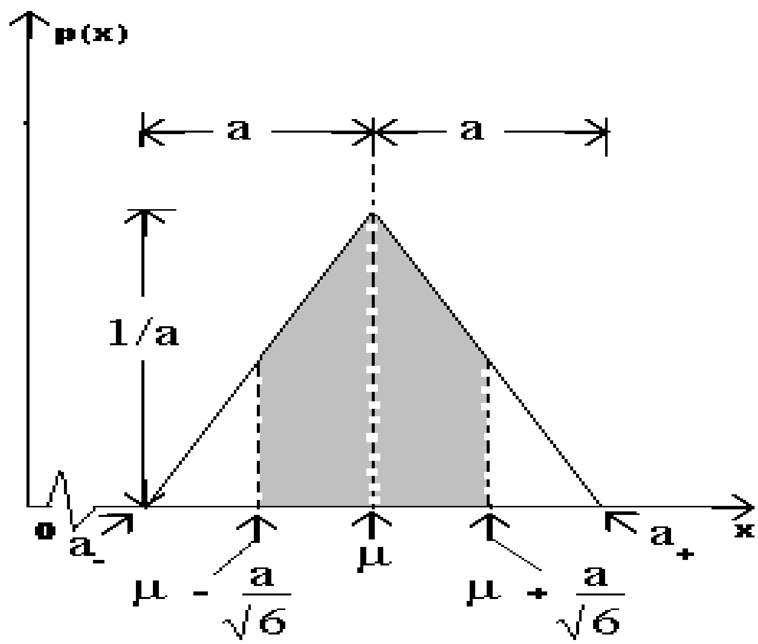

Figure A.3: Schematic view of the triangular distribution

A.4 U-Shaped Distribution

This U-shaped distribution is used in the case of mismatch uncertainty in radio and microwave frequency power measurements (shown in figure (A.4). At high frequency the power is delivered from a source to a load, and reflection occurs when the impedances do not match. The mismatch uncertainty is given by $2\Gamma_s\ \Gamma_L$ where Γ_s and Γ_L are the reflection coefficients of the source and the load respectively. The standard uncertainty is computed as:

$$u^2\ (x_i) = (2\ \Gamma_S\ \Gamma_L)^2\ /\ 2 \qquad \text{(A.11)}$$

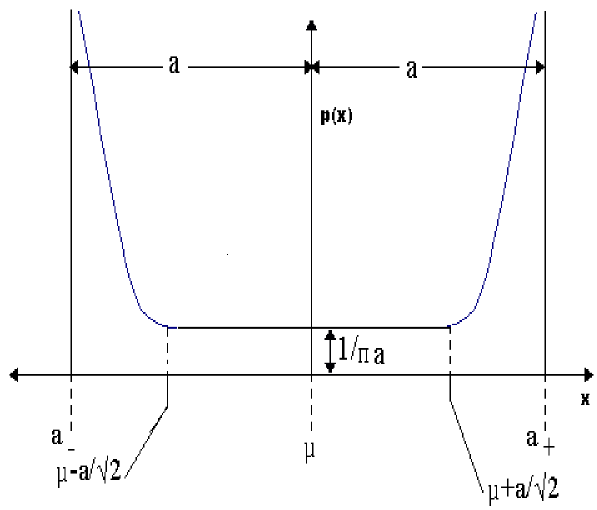

Figure A.4: Schematic view of the U-shaped distribution

Appendix B

Coverage factor derived from effective degrees of freedom

B.1 To estimate the value of a coverage factor k corresponding to a specified coverage probability requires that the reliability of the standard uncertainty u(y) of the output estimate y is taken into account. That means taking into account how well u(y) estimates the standard deviation associated with the result of the measurement. For an estimate of the standard deviation of a normal distribution, the degrees of freedom of the estimate, which depends on the size of the sample on which it is based, is a measure of the reliability. Similarly, a suitable measure of the reliability of the standard uncertainty associated with an output estimate is its effective degrees of freedom ν_{eff}, which is approximated by an appropriate combination of the ν_i for its different uncertainty contributions u_i (y).

B.2 The procedure for calculating an appropriate coverage factor k :

Step 1 Obtain the standard uncertainty associated with the output estimate.

Step 2 Estimate the effective degree of freedom ν_{eff} of the standard uncertainty u(y) associated with the output estimate y from the Welch-Satterthwaite formula.

Step 3 Obtain the coverage factor k from the table of values of student "t" distribution. If the value of ν_{eff} is not an integer, it is truncated to the next lower integer and the corresponding coverage factor k is obtained from the table.

B.3 Welch-Satterthwaite formula is as follows:

$$\nu_{eff} = \frac{u^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{\nu_i}} \qquad \text{(B.1)}$$

where u_i (y) (i = 1,2,3,……..N) defined in Eqs. (5.1) and (5.2) , are the contributions to the standard uncertainty associated with the output estimate y resulting from the standard uncertainty associated with the input estimate x_i which are assumed to be mutually statistically independent , and the ν_i is the effective degrees of freedom of the standard uncertainty contributions u_i (y).

Note :

The calculation of the degrees of the freedom ν for Type A and Type B of the evaluation may be as follows:

Type A Evaluation

For the results of direct measurement (Type A evaluation), the degree of freedom is related to the number of observations (n) as,

$$\nu_i = n - 1 \qquad \text{(B.2)}$$

Type B Evaluation

For this evaluation, when lower and upper limits are known

$$\nu_i \rightarrow \infty \qquad \text{(B.3)}$$

It is suggested that ν_i should always be given when Type A and Type B evaluations of uncertainty components are documented.

Where high precision measurements are undertaken, the accredited calibration laboratories shall be required to follow ISO Guide to the expression of uncertainty in Measurement (1995). Concerned laboratories should refer to Annexure – G (with special emphasis on table G-2) and Annexure –H for related examples.

However, assuming that $\nu_i \rightarrow \infty$ is not necessarily unrealistic, since it is a common practice to chose a_- and a_+ in such a way that the probability of the quantity lying outside the interval a_- to a_+ is extremely small.

Further interpretation on the above is given on page 29 & 30

Interpretation on Effective Degrees of Freedom

"Whilst the reason for determining the number of degrees of freedom associated with an uncertainty component is to allow the correct selection of value of student's t, it also gives an indication of how well a component may be relied upon. A high number of degrees of freedom is associated with a large number of measurements or a value with a low variance or a low dispersion associated with it. A low number of degrees of freedom corresponds to a large dispersion or poorer confidence in the value.

Every component of uncertainty can have an appropriate number of degrees of freedom, ν, assigned to it. For the mean, $\bar{x}$, for example $\nu = n - 1$, where n is a number of repeated measurements. For other Type A assessments, the process is also quite straightforward. For example, most spreadsheets provide the standard deviation of the fit when data is fitted to a curve. This standard deviation may be used as the uncertainty in the fitted value due to the scatter of the measurand values. The question is how to assign components evaluated by Type B processes.

For some distributions, the limits may be determined so that we have complete confidence in their value. In such instances, the number of degrees of freedom is effectively infinite. The assigning of limits, which are worst case, leads to this instance, namely infinite degrees of freedom, and simplifies the calculation of effective degrees of freedom of the combined uncertainty.

If the limits themselves have some uncertainty, then a lesser number of degrees of freedom must be assigned. The ISO Guide to the expression of uncertainty in measurement (GUM) gives a formula that is applicable to all distributions. It is equation G.3 that is:

$$\nu \approx \tfrac{1}{2} \left[\Delta u(x_i) / u(x_i) \right]^{-2} \qquad \text{......... .... 1}$$

Where :

$\Delta u(x_i)/ u(x_i)$ is the relative uncertainty in the uncertainty

This is a number less than 1, but may for convenience be thought of as a percentage or a fraction. The smaller the number, the better defined is the magnitude of the uncertainty.

For example, if relative uncertainty is 10%, i.e.

$$\Delta u(x_i)/ u(x_i) = 0.1$$

Then it can be shown that the number of degrees of freedom is 50. For a relative uncertainty of 25 % then $\nu = 8$ and for relative uncertainty of 50 %, ν = is only 2.

Rather than become seduced by the elegance of mathematics, it is better to try to determine the limits more definitely, particularly if the uncertainty is a major one. It is of the interest to note that equation (1) tells us that when we have made 51 measurements and taken the mean, the relative uncertainty in the uncertainty of the mean is 10%. This shows that even when many measurements are taken, the reliability of the uncertainty is not necessarily any better than when a type B assessment is make. Indeed, it is usually better to rely on prior knowledge rather than using an uncertainty based on two or three measurements. It also shows why we restrict uncertainty to two digits. The value is usually not reliable enough to quote to better than 1 % resolution.

Once the uncertainty components have been combined, it remains to find the number of degrees of freedom in the combined uncertainty. The degrees of freedom for each component must also be combined to find the effective number of degrees of freedom to be associated with the combined uncertainty. This is calculated using the Welch-Satterthwaite equation, which is:

$$\nu_{eff} = [u_c^4 (y) / \sum_{1}^{n} \{ u_i^4 (y) / \nu_i \}] \qquad \ldots 2$$

Where:

ν_{eff} is the effective number of degrees of freedom for u_c the combined uncertainty

ν_i is the number of degrees of freedom for u_i, the i^{th} uncertainty term

$u_i (y)$ is the product of $c_i u_i$

“The other terms have their usual meaning”.

Adopted from NATA document on “Assessment of uncertainties in Measurement”, 1999.

Table B.1: Student t-distribution for degrees of freedom ν. The t-distribution for ν defines an interval $-t_p(\nu)$ to $+t_p(\nu)$ that encompasses the fraction p of the distribution. For p = 68.27 %, 95.45 %, and 99.73 %, k is 1, 2, and 3, respectively.

Degrees Freedom	**Fraction p in percent**					
(ν)	**68.27**	**90**	**95**	**95.45**	**99**	**99.73**
1	1.84	6.31	12.71	13.97	63.66	235.80
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.90
7	1.08	1.89	2.36	2.43	3.50	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
11	1.05	1.80	2.20	2.25	3.11	3.85
12	1.04	1.78	2.18	2.23	3.05	3.76
13	1.04	1.77	2.16	2.21	3.01	3.69
14	1.04	1.76	2.14	2.20	2.98	3.64
15	1.03	1.75	2.13	2.18	2.95	3.59
16	1.03	1.75	2.12	2.17	2.92	3.54
17	1.03	1.74	2.11	2.16	2.90	3.51
18	1.03	1.73	2.10	2.15	2.88	3.48
19	1.03	1.73	2.09	2.14	2.86	3.45
20	1.03	1.72	2.09	2.13	2.85	3.42
25	1.02	1.71	2.06	2.11	2.79	3.33
30	1.02	1.70	2.04	2.09	2.75	3.27
∞	1.000	1.645	1.960	2.000	2.576	3.000

Appendix C

Solved examples showing the applications of the method outlined here to eight specific problems in different fields

C.1 Micrometer calibration using “0” grade slip gauge at 25mm

Introduction

The instrument under calibration is a micrometer of 0 – 25 mm range with a slip gauge of 25 mm of nominal size.

The detailed specifications of the slip gauge are as follows:

Range = 25 mm,

Standard reference temperature (T_{ref}) = 20 ^{0} C,

Actual calibrating temperature (T_c) = 23 ^{0} C,

Calibrated value = 25.00010 $\pm$ 0.00008 mm, and

Least count of thermometer used = 1 ^{0} C.

Mathematical model

$$Y_{GUT} = X_{STD} + \Delta X \qquad \text{(C.1)}$$

Where Y_{GUT} is the micrometer reading [Gauge under test (GUT), X_{STD} is the gauge block size and ΔX is the error or the difference between the micrometer reading and gauge block size.

Uncertainty equation

The combined standard uncertainty equation is given by,

$$u_c(Y_{GUT}) = \sqrt{\left[\left\{\frac{\delta Y_{GUT}}{\delta X_{STD}}\right\}\{u(X_{STD})\}\right]^2 + \left[\left\{\frac{\delta Y_{GUT}}{\delta \Delta X}\right\}\{u(\Delta X)\}\right]^2} \qquad \text{(C.2)}$$

Measured results

Type A evaluation

Five readings are taken and the deviation from the nominal value is as follows.

Mean Deviation:

$$\overline{x} = (\sum_{j=1}^{n} x_j)/n = 0.0006mm \tag{C.3}$$

Standard deviation :

$$s(x) = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_j - \overline{x}\right)^2} = \sqrt{\frac{1}{4}\left(12\times10^{-7}\right)} = \sqrt{3\times10^{-7}}\,mm = 5.47\times10^{-4}\,mm \tag{C.4}$$

Table C.1 : Data for calculation of mean and standard deviation

Observation numbers	Deviation from nominal value (x_j) (mm)	$\overline{x}$ (mm)	$(x_j - \overline{x})^2 \times 10^{-7}$ (mm)
1.	0.001		1.6
2.	0.000		3.6
3.	0.001	0.0006	1.6
4.	0.000		3.6
5.	0.001		1.6

Standard deviation of the mean:

$$s\left(\overline{x}\right) = \sqrt{\frac{s^2(x)}{n}} = \frac{5.47\times10^{-4}}{\sqrt{5}} = 2.446\times10^{-4}\,mm = 0.2446\mu m \tag{C.5}$$

Standard uncertainty:

$$u\left(\overline{x}\right) = 0.2446\mu m \tag{C.6}$$

Degrees of freedom (ν_i)

$$\nu = n - 1 = 5 - 1 = 4 \tag{C.7}$$

Type B evaluation

The uncertainty quoted in the gauge block calibration certificate is considered to be Type B uncertainty of normal distribution.

Standard uncertainty (u_1) due to the temperature measurement ±1 ^{0} C.

Standard thermal expansion coefficient of the gauge block is $11.5\times10^{-6}/\ ^0C$

$$u_1 = 25 \times 1 \times 11.5 \times 10^{-6}\ mm = 287.5 \times 10^{-6}\ mm = 0.2875\ \mu m \qquad (C.8)$$

Standard uncertainty (u_2) due to difference in temperature of micrometer and slip gauge

Assuming the temperature of the slip gauges and micrometer are the same but still it can have a difference ±1 ^{0} C. Hence, again uncertainty component

$$u_2 = 0.2875\ \mu m \qquad (C.9)$$

Standard uncertainty (u_3) due to difference in thermal expansion coefficient of the slip gauge and micrometer

It is assumed that the difference in thermal expansion coefficient of standard slip gauge and the micrometer screw is amounting to 20 %, hence the uncertainty component
[$\Delta T = T_c - T_{ref} = 3^0$ C],

$$u_3 = 25 \times 3 \times 11.5 \times 10^{-6} \times (20/100)\ mm = 0.1725\ \mu m \qquad (C.10)$$

Standard uncertainty (u_4) due to the flatness of micrometer faces'

$$say\left(1\frac{1}{2}\,fringe\right) \approx u_4 = 0.5\mu m \qquad (C.11)$$

Standard uncertainty (u_5) due to the parallelness of micrometer faces'

$$say\left(1\frac{1}{2}\,fringe\right) \approx u_5 = 0.5\mu m \qquad (C.12)$$

Standard uncertainty (u_6) due to the Standard used for calibration

The uncertainty in the value of the standard is taken from the calibration certificate say 0.08 µm. Assuming rectangular distribution, the standard uncertainty is

$$u_6 = \frac{0.08}{\sqrt{3}}\mu m = 0.046\mu m \qquad (C.13)$$

The sensitivity coefficients (c_i) are 1 and degree of freedom is $\nu_i = \infty$ [Type B components] in all six cases .

Degrees of freedom (ν_{eff})

$$\nu_{eff} = \frac{u_c(y)^4}{\sum_{i=1}^{n} \frac{(u_c(y))^4}{\nu_i}}$$

$$V_{eff} = \frac{(0.531)^4}{\frac{(0.2446)^4}{4} + \frac{(0.165)^4}{\infty} + \frac{(0.165)^4}{\infty} + \frac{(0.099)^4}{\infty} + \frac{(0.288)^4}{\infty} + \frac{(0.288)^4}{\infty} + \frac{(0.046)^4}{\infty}}$$

(C.14)

$$= \frac{0.07950}{\frac{0.00358}{4}}$$

$\cong$ 89 $\cong \infty$

Combined uncertainty

Combined uncertainty [u_c (Y_{GUT})] is ,

$$= \sqrt{(0.2446)^2 + (0.165)^2 + (0.165)^2 + (0.099)^2 + (0.288)^2 + (0.288)^2 + (0.046)^2}\ \mu m \quad \text{(C.15)}$$

u_c (Y_{GUT}) = 0.531 μm (C.16)

Table C.2: Uncertainty Budget:

Source of Uncertainty X_i	Estimates x_i (μm)	Limits $\pm \Delta x_i$ (μm)	Probability Distribution – Type A or B - factor	Standard Uncertainty $u(x_i)$ (μm)	Sensitivity coefficient c_i	Uncertainty contribution $u_i(y)$ (μm)	Degree of freedom ν_i
u_1	0.287	0.143	Rectangular - Type B - √3	0.165	1.0	0.165	∞
u_2	0.287	0.143	Rectangular - Type B - √3	0.165	1.0	0.165	∞
u_3	0.172	0.086	Rectangular - Type B - √3	0.099	1.0	0.099	∞
u_4	0.5	0.25	Rectangular - Type B - √3	0.288	1.0	0.288	∞
u_5	0.5	0.25	Rectangular - Type B - √3	0.288	1.0	0.288	∞
u_6	0.08	0.04	Rectangular - Type B - √3	0.046	1.0	0.046	∞
Repeatability	$u(\bar{x})$		Normal - Type A - √5	0.547	1.0	0.2446	4
$u_c(Y_{GUT})$						0.531	∞
Expanded uncertainty			k = 2			1.062	∞

Expanded Uncertainty (U)

From the student's distribution table, for the confidence level of 95.4 % and for $\nu_{eff} = \infty$, the coverage factor k = 2.

$$U = k\, u_c(Y_{GUT}) = 2 \times 0.531\ \mu m = 1.062\ \mu m$$

Reporting of results

The value at 25 mm is

25.00010 mm ± 1.062 μm

with coverage factor k = 2 for confidence level of 95.4 % and for $\nu_{eff} = \infty$.

C.2 Calibration of weight of nominal value 10 kg

Introduction

The calibration of a weight of nominal value 10 kg of OIML class M1 is carried out by comparison to a reference standard (OIML class F2) of the same nominal value using a mass Comparator whose performance characteristics have previously been determined.

Mathematical model

The unknown conventional mass m_x, is obtained as

$$m_x = m_s + \delta m_D + \delta m + \delta m_c + \delta A \qquad \text{(C.17)}$$

where m_s - conventional mass of the standard,
δm_D - drift of the value of the standard since its last calibration,
δm - observed difference in mass between the unknown mass and the standard
δm_c - correction for eccentricity and magnetic effects,
δA - correction for air buoyancy.

For uncorrelated input quantities, the combined standard uncertainty is given by

$$u_c^2(y) = \sum_{i=1}^{n} \left[\frac{\delta f}{\delta x} \right]^2 u^2(x_i) \qquad \text{(C.18)}$$

Details of the specifications

1. Reference standard (m_s): The calibration certificate for the reference standard gives a value of 10,000.005 g with an associated expanded uncertainty of 45 mg (coverage factor k = 2)

2. Drift of the value of the standard (δm_p): The drift of the value of the reference standard is estimated from the previous calibrations to be zero within ± 15 mg.

3. Comparator (δm): A previous evaluation of the repeatability of the mass difference between two weights of the same nominal value gives a pooled estimate of standard deviation of 25 mg.

4. Eccentricity and magnetic effect (δm_c): The variation of mass due to eccentric load and magnetic effect is found to be ± 10 mg.

5. Air buoyancy (δ_A): The limit of air buoyancy correction is found to be 10 mg.

Table C.3 : Observations

No.	A reading (g)	B reading (g)	B reading (g)	A Reading (g)	Observed difference (g)
1.	0.015	0.020	0.025	0.010	0.010
2.	0.010	0.030	0.020	0.010	0.015
3.	0.020	0.045	0.040	0.015	0.025
4.	0.020	0.040	0.030	0.010	0.020
5.	0.010	0.030	0.020	0.010	0.015

1

Type A evaluation

Five observations of the difference in mass between the unknown mass (B) and the standard (A) are obtained using the substitution method and the ABBA weighing sequences:
Arithmetic mean $\bar{\delta} m$ = 0.017 g,

pooled estimate of standard deviation s_p (δ m) = 0.025 g (obtained from prior evaluation)

Standard uncertainty u_A = u(δm) = s($\bar{\delta}$m) $= \frac{25}{\sqrt{5}} mg$ = 11.18 mg,

Degrees of freedom = 5 – 1 = 4

Type B evaluation

1. From the calibration certificate of the standard (A), the expanded uncertainty (U) is certified as 45 mg with a coverage factor k = 2. the standard uncertainty

$$u(m_s) = \frac{45}{2} mg = 22.5 mg$$

2. Drift in the value of the standard is quoted as ±15 mg. Assuming a rectangular distribution, the standard uncertainty

$$u(\delta m_D) = \frac{15}{\sqrt{3}} mg = 8.66 mg$$

3. Due to eccentricity and magnetic effects, the variation of mass is found to be ± 10 mg. Assuming a rectangular distribution, the standard uncertainty,

$$u(\delta m_c) = \frac{10}{\sqrt{3}} mg = 5.77 mg$$

4. Error in air buoyancy correction is reported to be ±10 mg. Assuming rectangular distribution, the standard uncertainty

$$u(\delta A) = \frac{10}{\sqrt{3}} mg = 5.77 mg$$

$$u_B = \sqrt{(22.5)^2 + (8.66)^2 + (5.77)^2 + (5.77)^2} mg = 25.45 mg \quad \text{(C.19)}$$

Degrees of freedom (ν_i): In all these four cases, the degree of freedom is $\nu_i = \infty$

Degrees of freedom ν_{eff}

$$\nu_{eff} = \frac{(27.8)^4}{\frac{(11.8)^4}{4} + \frac{(22.5)^4}{\infty} + \frac{(8.66)^4}{\infty} + \frac{(5.77)^4}{\infty} + \frac{(5.77)^4}{\infty}}$$

$$= \frac{598029.44}{\frac{15623.1}{4}}$$

$$\simeq 153.1 \simeq \infty$$

Combined standard uncertainty

Combined standard uncertainty is

$$u_c = \sqrt{u_A^2 + u_B^2} = \sqrt{(11.18)^2 + (25.45)^2} = 27.8 mg \quad \text{(C.20)}$$

Expanded uncertainty

U = ku_c (m) = 2 × 27.8 mg = 55.6 mg

Reported result

The measured mass of the nominal 10 kg weight is 10.000025 kg ± 56 mg. The reported expanded uncertainty of measurement is stated as the standard uncertainty in measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability 99.5 %.

Table C.4: Uncertainty Budget:

Source of Uncertainty X_i	**Estimates x_i**	**Limits $\pm \Delta x_i$**	**Probability Distribution -Type A or B - Factor**	**Standard Uncertainty $u(x_i)$** (mg)	**Sensitivity coefficient c_i**	**Uncertainty contribution $u_i(y)$** (mg)	**Degree of freedom v_i**
m_s	10.000005 kg	45 mg	Normal - Type B - 2	22.5	1.0	22.5	∞
δm_d	15 mg	7.5 mg	Rectangular - Type B - √3	8.66	1.0	8.66	∞
δm_c	10 mg	5 mg	Rectangular - Type B - √3	5.77	1.0	5.77	∞
δA	10 mg	5 mg	Rectangular - Type B - √3	5.77	1.0	5.77	∞
Repeatability			Normal - Type A - √5	11.18	1	11.18	4
u_c (m)						27.8	∞
Expanded uncertainty			k = 2			55.6	∞

C.3 Calibration of a Industrial Dead Weight Tester

An industrial Dead Weight Tester (DWT) up to 60 MPa is calibrated against a standard piston cylinder (SPC) assembly, which is used as the secondary standard. The specifications of the industrial DWT and SPC are as follows:

Dead Weight Tester

Range: 0.1 – 60 MPa
Serial No: XXXXXXX
Make: XXXXXXXX

Manufacturer's data :

Resolution - $\pm$ = 0.1 MPa
Accuracy - $\pm$ 0.1 % of full scale pressure

SECONDARY STANDARD – STANDARD PISTON CYLINDER ASSEMBLY

Range — 0.2 – 100 MPa
Class — S

The data obtained from the characterization of the piston gauge is as follows:

Resolution - $\pm$ 0.01 MPa
Standard uncertainty – $\pm$ 0.01 MPa in the whole studied range of pressure

Calibration Procedure

The calibration is carried out by crossfloat method. This method starts with the following steps:

Leveling: - the instrument has been leveled so that the axis of rotation of the piston is vertical.

Rotation: - during the calibration, the piston of the secondary standard gauge (SPC) as well as the industrial DWT have been rotated at a constant rpm with a synchronous motor to relieve friction.

Temperature: - the temperature of SPC and DWT has been maintained near 23^0C and suitable temperature correction has been incorporated to make all the results at the same temperature, that is, at 23 ^{0}C.

Repetition: - the calibrations have been carried out at least five times under identical condition both at the increasing pressure as well as decreasing pressure.

Acceleration due to gravity (g) correction: - The correction has been incorporated whenever there is a difference in g- value. For example, g_{NPL} is 9.791241 m/s^2 therefore the correction factor is ($9.791241/g_{Manufac}$).

Reference level: - The pressure generated at the reference level both at the secondary standard (SPC) and test gauge (DWT) has been maintained constant.

Mathematical Model

The mathematical relationship can be modeled as:

$$P_{DWT} = \overline{P_{SPC}} + \Delta P,$$

Where P_{DWT} is the pressure as measured by the industrial DWT , $\overline{P_{SPC}}$ is the average standard pressure as obtained from the standard gauge P_{SPC} . The average sign indicate the arithmetic mean of several repetitive measurement under identical condition. ΔP is the difference between the two readings of the P_{DWT} and the $\overline{P_{SPC}}$. Table (C.4) represents the data where σ represents the standard uncertainty in each set of readings at a given pressure.

In the simplest case and also in this limited pressure range up to 100 MPa, it is normally observed that the ΔP is a linear function of $\overline{P_{SPC}}$, which is,

$$\Delta P = \Delta P_o + S_1 \times \overline{P_{SPC}} , \qquad (C.22)$$

where ΔP_0 and S_1 are assumed to be constants ,

The combined standard uncertainty equation is then given by

$$u_c(P_{DWT}) = \sqrt{\left[\left\{\frac{\delta P_{DWT}}{\delta P_{SPC}}\right\}\{u(P_{SPC})\}\right]^2 + \left[\left\{\frac{\delta P_{DWT}}{\delta \Delta P_O}\right\}\{u(\Delta P_O)\}\right]^2 + \left[\left\{\frac{\delta P_{DWT}}{\delta S_1}\right\}\{u(S_1)\}\right]^2} \qquad (C.23)$$

where the bracketed quantities are the standard uncertainties due to the repeatability of the readings and the standard gauge, respectively. It may be mentioned here that the partial derivatives of P_{DWT} with P_{SPC} and P_{DWT} with ΔP_o is 1 , therefore, the sensitivity coefficients also for both these cases equal to one. However partial derivative of P_{DWT} with S_1 is $\overline{P_{SPC}}$. Therefore, we have to take into account the value of $\overline{P_{SPC}}$ for the estimation of combined standard uncertainty.

Experimental Results

Table (C.4) shows the data as obtained from the experiment. As mentioned, we have taken five readings (n = 5) of the same pressure point while increasing / decreasing the pressure cycle. The $\overline{P_{SPC}}$ is the average of these five readings. ΔP is the error or the difference between P_{DWT} and $\overline{P_{SPC}}$. PW represents Piston and Weight hanger.

Type A evaluation of standard uncertainty

(1) Repeatability

We have taken 5 repeatable readings at each and every pressure or n= 5. The maximum standard uncertainty (σ) from the table (C.5) is,

$$s(q_k) = 0.0026827 \text{ MPa} \quad \text{(C.24)}$$

Hence, the standard uncertainty is given by,

$$u_1(s) = 0.0026827/\sqrt{5} = 0.001199 MPa \quad \text{(C.25)}$$

Degree of freedom ν_i = 5 - 1 = 4

Table C.5: Increasing and Decreasing Pressure

Weight used	Nominal Pressure P_{DWT} (MPa)	Reading of SPC (MPa)					$\overline{P_{SPC}}$ (MPa)	σ (kPa)	ΔP (kPa)
PW	1.0	0.9989	0.9992	0.9988	0.9978	0.9991	0.9987	0.619	1.24
PW , 7-8	5.0	4.9965	4.9967	4.9964	4.9971	4.9961	4.9965	0.422	3.44
PW,1	10.0	9.9931	9.993	9.9933	9.9931	9.9935	9.9932	0.162	6.80
PW 1-2	20.0	19.9861	19.9865	19.9867	19.9871	19.9861	19.9865	0.477	13.50
PW , 1-3	30.0	29.9791	29.9787	29.9782	29.9799	29.9787	29.9789	0.728	21.08
PW,1-4	40.0	39.9694	39.9706	39.971	39.9701	39.9691	39.9700	0.896	29.96
PW, 1-4 , 6	45.0	44.9665	44.9654	44.9659	44.9671	44.9669	44.9663	0.675	33.64
PW , 1-5	50.0	49.96	49.9611	49.9606	49.9604	49.9595	49.9603	0.626	39.68
PW , 1-6	55.0	54.9548	54.9576	54.9556	54.9561	54.9545	54.9557	1.364	44.28
PW, 1-9	60.0	59.9494	59.9532	59.9498	59.9512	59.9532	59.9513	1.819	48.64
PW , 1-9	60.0	59.9497	59.9552	59.9549	59.9512	59.9532	59.9528	2.682	47.16
PW , 1-6	55.0	54.9568	54.9576	54.9556	54.9561	54.9545	54.9561	0.780	43.88
PW, 1-5	50.0	49.9641	49.9611	49.9606	49.9604	49.9595	49.9611	0.911	38.86
PW, 1-4 , 6	45.0	44.9675	44.9654	44.9659	44.9671	44.9669	44.9665	1.000	33.44
PW , 1-4	40.0	39.9704	39.9706	39.971	39.9701	39.9691	39.9702	0.603	29.76
PW, 1-3	30.0	29.9799	29.9787	29.9782	29.9801	29.9787	29.9791	0.943	20.88
PW , 1-2	20.0	19.9871	19.9875	19.9867	19.9871	19.9861	19.9869	0.425	13.10
PW , 1	10.0	9.9939	9.993	9.9933	9.9931	9.9935	9.9933	0.392	6.64
PW , 7-8	5.0	4.9975	4.9967	4.9964	4.9971	4.9961	4.9967	0.467	3.24
PW	1.0	0.9997	0.9992	0.9988	0.9978	0.9991	0.9989	0.116	1.08

(2) Data Analysis

As mentioned in the mathematical model, there is a difference in pressure (ΔP) at each pressure of $\overline{P_{SPC}}$ [(Eq. (C.22)] . Thus, this ΔP can be fitted with $\overline{P_{SPC}}$ in a linear fitting program.

In the present case, we have 20 data points as are shown in Table (C.5), the fitted equation reduces to,

$$\Delta P = 0.000802 \times \overline{P_{SPC}}(MPa) - 0.0013(MPa) \quad \text{(C.26)}$$

The different fitting parameters with standard uncertainty are shown in Table (C.6). From the above Eq.(C.26), ΔP is equal to 0.04682 MPa at 60 MPa. It is therefore ΔP is maximum at 60 MPa but reduces as we decrease the pressure.

Table C.6: Regression Output:

Source Parameters	**Fitted Value**
ΔP_o	-0.001304 (MPa)
$U(\Delta P_o)$ (1 σ)	0.0012762 (MPa)
S_1	0.000802
$u(S_1)$ (1 σ)	0.000014
Degrees of Freedom	19

The standard uncertainty in ΔP is evaluated from Eq. (C.23) with the sensitivity coefficient, as mentioned earlier, equals to $\overline{P_{SPC}}$.

$$u_2(\Delta P) = \sqrt{u(\Delta P_o)^2 + \left(\overline{P_{SPC}}\right) \times u(S_1)^2} \quad \text{(C.27)}$$

At the maximum pressure [60 MPa], the standard uncertainty reduces to,

$$u_2(\Delta P) = \sqrt{(0.0012762)^2 + (60 \times 0.000014)^2}$$

$$= 0.001527 \text{ MPa} \quad \text{(C.28)}$$

Therefore, estimated Type A standard uncertainty is

$$u_A = \sqrt{u_1(s)^2 + u(\Delta P)^2}$$

$$= \sqrt{(0.001199)^2 + (0.001527)^2} = 0.00194MPa \quad \text{(C.29)}$$

Type B Evaluation of Standard uncertainty

From the specifications of the standard piston cylinder assembly, there is an equal probability of the value lying anywhere between the lower (a_-) and upper (a_+) limits, the boundaries of this rectangular distribution is given by

$$a = \frac{a_+ - a_-}{2} = 0.01 MPa \qquad (C.30)$$

Hence, the Type B component is given by,

$$u_B = \frac{a}{\sqrt{3}} = \frac{0.01}{\sqrt{3}} = 0.0057 MPa \qquad (C.31)$$

Degree of freedom $\nu_i = \infty$

Table C.7: Summary of standard uncertainty components

Source of uncertainty (X_i)	**Estimates (x_i) (MPa)**	**Limits $\pm\Delta x_i$ (MPa)**	**Probability Distribution –Type A or B - factor**	**Standard uncertainty $u(x_i)$ (MPa)**	**Sensitivity coefficient**	**Uncertainty contribution u_i (y) (MPa)**	**Deg. of freedom (ν_i)**
u_B	0.01	0.01	Rectangular - Type B $\sqrt{3}$	0.00570	1.0	0.0057	∞
u_A			Normal -Type A	0.00194	1.0	0.00194	19
u_c (P_{DWT})						0.00602	∞
Expanded uncertainty			k = 2.00			0.012	∞

Combined standard uncertainty

The combined standard uncertainty is then given by

$$u_c(P_{DWT}) = \sqrt{u_A^2 + u_B^2} \qquad (C.32)$$

$$= \sqrt{(0.00194)^2 + (0.0057)^2} MPa = 0.00602 MPa \qquad (C.33)$$

Effective Degree of freedom (ν_{eff})

The effective degree of freedom of the combined standard uncertainty is given by

$$\nu_{eff} = \frac{(u_c)^4}{\frac{u_1^4}{\nu_1} + \frac{u_2^4}{\nu_1} + \frac{u_B^4}{\infty}}$$

$$= \frac{(0.00602)^4}{\frac{(0.001199)^4}{4} + \frac{(0.001527)^4}{19} + \frac{(0.0057)^4}{\infty}}$$

$$\approx \quad \infty \qquad \text{(C.34)}$$

Expanded uncertainty

Using the student's t-distribution table, k = 2.0 for a confidence level of approximately 95.45%. Therefore, the expanded uncertainty is given by

$$U = k \times u_c\ (P_{DWT})$$

$$= 2.00 \times 0.00602 \text{ MPa}$$

$$\approx 0.012 \text{ MPa}$$

Reporting of results

For the range 0- 60 MPa, the uncertainty U is $\pm$ 0.012 MPa which is approximately 0.02 % of the full scale pressure. This is determined from a combined standard uncertainty u_c = 0.00602 MPa and a coverage factor k = 2.00 based on students distribution for $\nu = \infty$ degrees of freedom and estimated to have a level of confidence of 95.45 %.

C.4 Estimation of measurement uncertainty in luminous flux measurement

For Luminous flux measurement of light sources with integrating sphere, a substitution method is applied in which a test lamp substitutes a luminous flux standard and the luminous flux of a test source is evaluated by comparing the indirect illuminance in the two cases.

The luminous flux measurement has to be conducted as follows:

1. Switch on the measuring equipment and let the auxiliary lamp warm up for 15 minutes.
2. Mount the standard lamp into the sphere center.
3. After burning –in period, the indirect illumination by standard lamp E_s is measured.
4. Turn the supply voltage down and switch off the standard lamp.
5. The switched on auxiliary lamp is moved into the sphere. It should remain switched on always to avoid warm up period. The indirect illuminance E_{AS} is measured.
6. The standard lamp is moved out of the sphere and the test lamp to be measured is mounted into the sphere center with the auxiliary lamp still burning, the indirect illuminance E_{AT} is measured.
7. Turn on the test lamp to be measured. After burning in period the indirect illuminance E_T is measured.

From the above it is clear that the luminous flux of the test lamp is a function of the luminous flux of the standard lamp Φ_S, the indirect illuminance from the auxiliary lamp E_{AS} and E_{AT} with standard lamp inside the integrating sphere and test lamp inside the sphere respectively and the indirect illuminance E_S and E_T produced by the standard lamp and the test lamp respectively. The functional dependence of Φ_T, can be written as

$$\Phi_{T,} = f\,(\Phi_S, E_S, E_T, E_{AS}, E_{AT}) \qquad \text{(C. 35)}$$

The luminous flux Φ_T, of the lamp can be calculated from the luminous flux Φ_S of the standard lamp according to the following relation

$$\Phi_{T,} = \Phi_s \times \frac{E_s}{E_T} \times \left[\frac{E_{AS}}{E_{AT}}\right] \qquad \text{(C.36)}$$

The factor E_{AS}/E_{AT} considers the effect of different sizes and types of test lamps and the standard lamps.

Since the quantities on the RHS of the Eq. (C.36) are in product form, the equation for the combined standard uncertainty can be expressed as an estimated relative combined variances

$$\frac{u^2(\Phi_T)}{\Phi_T^2} = \frac{u^2(\Phi_S)}{\Phi_S^2} + \frac{u^2(E_S)}{E_S^2} + \frac{u^2(E_T)}{E_T^2} + \frac{u^2(E_{AS})}{E_{AS}^2} + \frac{u^2(E_{AT})}{E_{AT}^2} \qquad \text{(C.37)}$$

If Φ_S for standard source is given to be 1045 ℓm and the standard uncertainty is $\pm$ 9.12 ℓm, from the measurement of expectation value of E_S / E_T and E_{AS} / E_{AT}, the value Φ_T can be calculated from Eq. (C.36).

Table C.8: Observations

S. No.	E_{AS}	$\overline{E_{AS}}$	$(E_{AS} - \overline{E_{AS}})^2$
1.	10.296		400×10^{-6}
2.	10.279		9×10^{-6}
3.	10.254		484×10^{-6}
4.	10.290		196×10^{-6}
5.	10.271	10.276	25×10^{-6}
6.	10.272		16×10^{-6}
7.	10.286		100×10^{-6}
8.	10.285		81×10^{-6}
9.	10.254		484×10^{-6}
10.	10.277		441×10^{-6}

For identical standard and test lamp of identical size, shape, same electrical parameters and same colour temperature, the expectation value of E_S / E_T and E_{AS} / E_{AT} will be close to 1. Hence the value of Φ_T will be almost equal to Φ_S and the uncertainty u (Φ_T) can be calculated from Eq. (C.37). However, we will calculate value of Φ_T and standard uncertainty in u (Φ_T) by the following example in which the uncertainties in the measurements of E_{AS} and E_{AT} and E_S and E_T are calculated by statistical method and is an example of Type A evaluation of standard uncertainty.

In the case study , for identical standard lamp and test lamp of identical shape and size , if the expectation or the average values of the E_{AS} , E_{AT} , E_S and E_T are 10.276 lux , 10.20 lux, 81.14 lux and 83.76 lux , respectively , the value of Φ_T from Eq. (C.36) comes out to be

$$\Phi_T = 1045 \times \frac{10.276}{10.20} \times \frac{83.76}{81.14} = 1086.6\ \ell m \qquad \text{(C.38)}$$

Type A Evaluation

We will illustrate by an example the calculation of the standard uncertainty u(E_{AS}) and relative standard uncertainty u(E_{AS}) / E_{AS} for one of the parameters, e.g. E_{AS} by Type A evaluation.

$$\sum_{i=1}^{10}\left(E_{AS}-\overline{E_{AS}}\right)^2 = 2326\times 10^{-6} \quad \text{(C.39)}$$

Variance

$$s^2(E_{AS}) = \sum_{i=1}^{10}\frac{\left(E_{AS}-\overline{E}_{AS}\right)^2}{9} = \frac{2326\times 10^{-6}}{9} \quad \text{(C.40)}$$

Standard Deviation

$$s(E_{AS}) = 16.08 \times 10^{-3} \quad \text{(C.41)}$$

Standard deviation of the mean which is known as standard uncertainty is

$$u\left(\overline{E_{AS}}\right) = \frac{16.08\times 10^{-3}}{\sqrt{10}} = 5.08\times 10^{-3} \quad \text{(C.42)}$$

The relative standard uncertainty is

$$\frac{u\left(\overline{E_{AS}}\right)}{\overline{E}_{AS}} = 5\times 10^{-4} \quad \text{(C.43)}$$

The degree of freedom in this case is

$$\nu_i = n - 1 = 10 - 1 = 9 \quad \text{(C.44)}$$

Similarly the relative uncertainties for E_{AT}, E_S and E_T, can be evaluated by Type A evaluation. The degree of freedom in each case is also 9 as the total number of observations made in each case are 10.

The values of the relative uncertainties are

$\frac{u\left(\overline{E_{AT}}\right)}{\overline{E}_{AT}}$ = 6 × 10^{-4} and degree of freedom in this case is = ν_i = n - 1 = 10 – 1 = 9

$\frac{u\left(\overline{E_S}\right)}{\overline{E}_S}$ = 1.2 × 10^{-2} and degree of freedom in this case is = ν_i = n - 1 = 10 – 1 = 9

$\frac{u\left(\overline{E_T}\right)}{\overline{E}_T}$ = 1.6 × 10^{-2} and degree of freedom in this case is = ν_i = n - 1 = 10 – 1 = 9

Type B Evaluation

The uncertainty in the value of Φ_S is $\pm$ 9.12 ℓm and is taken from the certificate of the calibration of the standard lamp. Assuming rectangular distribution, the standard uncertainty in the value of Φ_S is :

$$u(\Phi_s) = \frac{9.12}{\sqrt{3}} = 5.3\ell m \qquad \text{(C.45)}$$

degree of freedom in this case is $\nu_i = \infty$

The relative standard uncertainty is

$$\frac{u(\Phi_s)}{\Phi_s} = \frac{5.3}{1045} = 5 \times 10^{-3} \qquad \text{(C.46)}$$

Combined standard uncertainty

The value of the relative combined uncertainty $\frac{u_c(\Phi_T)}{\Phi_T}$ for the value of Φ_T is calculated using Eq. (C.37).

$$\frac{u_c(\Phi_T)^2}{\Phi_T^2} = 25 \times 10^{-6} + 144 \times 10^{-6} + 256 \times 10^{-6} + 0.36 \times 10^{-6} + 0.25 \times 10^{-6} \qquad \text{(C.47)}$$

Therefore, $\frac{u_c(\Phi_T)}{\Phi_T} = 2.06 \times 10^{-2}$

Effective degrees of freedom

$$\nu_{eff} = \frac{\left[\frac{u_c(\Phi_T)}{\Phi_T}\right]^4}{\frac{\left[\frac{u(\Phi_S)}{\Phi_S}\right]^4}{\infty} + \frac{\left[\frac{u(E_S)}{E_S}\right]^4}{9} + \frac{\left[\frac{u(E_T)}{E_T}\right]^4}{9} + \frac{\left[\frac{u(E_{AS})}{E_{AS}}\right]^4}{9} + \frac{\left[\frac{u(E_{AT})}{E_{AT}}\right]^4}{9}} \qquad \text{(C.48)}$$

$$\nu_{eff} = \frac{\left[2.06 \times 10^{-2}\right]^4}{\frac{\left[5 \times 10^{-3}\right]^4}{\infty} + \frac{\left[1.2 \times 10^{-2}\right]^4}{9} + \frac{\left[1.6 \times 10^{-2}\right]^4}{9} + \frac{\left[6 \times 10^{-4}\right]^4}{9} + \frac{\left[5 \times 10^{-5}\right]^4}{9}} \quad \text{(C.49)}$$

$$\nu_{eff} = 18.8 = 19 \quad \text{(C.50)}$$

Expanded uncertainty (U)

From Student's t distribution, for confidence level of 95 %, and for ν_{eff} = 18.8 = 19 , the coverage factor k = 2.09.

Reporting results

The value of ϕ_T = (1086.6 ± 22.4) ℓm and expanded uncertainty with coverage factor k = 2.09, the value of ϕ_T = (1086.6 ± 46.8) ℓm

Table C.9: Details of the uncertainty budget

Source of Uncertainty X_i	**Estimates x_i**	**Limits $\pm\Delta x_i$**	**Probability Distribution - Type A or B - Factor**	**Standard Uncertainty $u(x_i)$**	**Sensitivity coefficient c_i**	**Uncertainty contribution $u_i(y)$**	**Deg. of freedom ν_i**
E_S	$u(E_S)$		Normal - Type A - √10	1.2×10^{-2}	1.0	1.2×10^{-2}	9
E_T	$u(E_T)$		Normal - Type A - √10	1.6×10^{-2}	1.0	1.6×10^{-2}	9
E_{AS}	$u(E_{AS})$		Normal - Type A - √10	5×10^{-3}	1.0	5×10^{-3}	9
E_{AT}	$u(E_{AT})$		Normal - Type A - √10	1.6×10^{-3}	1.0	1.6×10^{-2}	9
Φ_S	$u(\Phi_S)$		Rectangular - Type B	5.3 *l*m	1.0	5.3 ℓm	∞
Combined uncertainties	$u_c(\Phi_T)$					22.4 ℓm	19
Expanded uncertainties			k = 2.09			46.8 ℓm	19

C.5 Temperature measurement using thermocouple

Introduction

A digital thermometer with a Type K thermocouple was used to measure the temperature inside a temperature chamber. The temperature controller of the chamber was set at 500^0C .

Digital thermometer specification

Resolution: 0.1^0 C
Type K accuracy (one year): $\pm$ 0.6 ^{0} C

Thermocouple

The Type K thermocouple is calibrated every year. The last calibration report provided an uncertainty of $\pm$ 2.0 ^{0} C at confidence level of 99 %. The correction for the thermocouple at 500 ^{0} C is 0.5^0 C.

Measurement record

When the temperature chamber indicator reached 500 ^{0} C, the readings were taken after a stabilization time of half an hour. Ten measurements were taken as recorded in Table (C.10)

Mathematical Model

The mathematical model is represented as follows:

$$T = D + Correction \qquad (C.51)$$

Where

T= Temperature measured.
D = Displayed temperature of the digital thermometer.
Correction = Correction due to the digital thermometer and Type K thermocouple

Uncertainty evaluation

The combined standard uncertainty (u_c) includes uncertainties of the repeatability of the displayed readings, the digital thermometer and the thermocouple. This can be represented in the equation below:

$$u_C = \sqrt{u_1^2 + u_2^2 + u_3^2} \qquad (C.52)$$

where, u_c = combined standard uncertainty in the measurement,
u_1 = standard uncertainty in the repeatability of measured readings,
u_2 = standard uncertainty in the digital thermometer,
u_3 = standard uncertainty in the thermocouple

Table C.10: Measurements record

Measurement (i)	T in ^{0} C
1	500.1
2	500.0
3	501.1
4	499.9
5	499.9
6	500.0
7	500.1
8	500.2
9	499.9
10	500.0

Analysis of measurement uncertainty components

Type A evaluation

(A) Standard uncertainty in the readings (u_1)

Table (C.10) shows the data obtained from the experiment. Mean value,

$$\overline{T} = \frac{1}{10}\sum_{i=1}^{10} T_i = 500.02 \qquad \text{(C.53)}$$

where T_i are the 10 measurements taken as listed in Table (C.10). The temperature of the chamber after taking into consideration the correction of the thermocouple is 500.5^{0}C.

The variance is calculated as follows:

$$s^2(T_i) = \frac{1}{n-1}\sum_{i=1}^{n}\left(T_i - \overline{T}\right)^2 = \frac{1}{9}(0.096) = 0.0106 \ {}^0C^2 \qquad \text{(C.54)}$$

Standard deviation

$$s\ (T_i) = 0.103\ ^0\ C \qquad \text{(C.55)}$$

Standard deviation of the mean is as follows:

$$u_1 = s(\overline{T}) = \frac{s(T_i)}{\sqrt{n}} = \frac{0.103\ ^0C}{\sqrt{10}} = 0.03^0\,C \qquad \text{(C.56)}$$

Thus the standard uncertainty (u_1) is equal to 0.03 ^{0} C.

Degrees of freedom (ν_i) = n - 1 = 10 – 1 = 9.

Type B evaluation

Standard uncertainty (u_2)

From specifications, the uncertainty in the digital thermometer is ± 0.6^0 C. Assuming rectangular distribution, the standard uncertainty in the digital thermometer (u_2) is,

$$u_2 = \frac{0.6}{\sqrt{3}} = 0.35\ {}^0C \quad \text{(C.57)}$$

degree of freedom (ν_i) = ∞

Standard uncertainty (u_3)

From calibration report, with a confidence level of 99 % (k = 2.58), the uncertainty in the thermocouple is ± 2.0 ^{0} C.

$$u_3 = \frac{2.0}{2.58} = 0.78^0 C \quad \text{(C.58)}$$

degrees of freedom (ν_i) = ∞

Combined standard uncertainty

The value of the combined standard uncertainty is calculated using Eq. (C.52)

$$u_c = \sqrt{(0.03)^2 + (0.35)^2 + (0.78)^2} = 0.85\ {}^0C \quad \text{(C.59)}$$

Effective degrees of freedom

$$\nu_{eff} = \frac{(0.85)^4}{\frac{(0.03)^4}{9}} = \infty \quad \text{(C.60)}$$

Expanded uncertainty

$$U = k \times u_c = 2 \times 0.85 = 1.7\ {}^0 C \quad \text{(C.61)}$$

Table C.11 : Statement of the uncertainty budget

Source of Uncertainty **X_i**	**Estimates x_I** **^{0}C**	**Limits $\pm\Delta x_I$** **^{0}C**	**Probability Distribution** - **Type A or B** - **Factor**	**Standard Uncertainty $u(x_i)$** **^{0}C**	**Sensitivity coefficient c_i**	**Uncertainty contribution $u_i(y)$** **^{0}C**	**Deg. of freedom ν_i**
Digital Thermometer	0.6	0.3	Rectangular - Type B - $\sqrt{3}$	0.35	1.0	0.35	∞
Thermocouple	2.0		Normal - Type B - 2.58	0.78	1.0	0.78	∞
Repeatability			Normal - Type A - $\sqrt{10}$	0.03	1.0	0.03	9
Combined uncertainty	u_c					0.85	∞
Expanded uncertainties	U		k = 2			1.7	∞

Reporting of results

The temperature of the chamber was measured to be 500.5 ^{0} C. The measurement uncertainty is ± 1.7 ^{0} C. The reported measurement uncertainty is estimated at a level of confidence of approximately 95 % with a coverage factor k of 2.

C.6 Calibration of a 6 ½ digit DMM on its 1 Volt AC range

Introduction

We discuss the method of calibration of a 6 ½ digit DMM on its 1 Volt AC range at a nominal 0.5 V level at 1 kHz using 0.5 V calibrated thermal voltage converter (TVC) . AC voltage from highly stable AC Calibrator is applied to both DMM (DUC) and the standard (TVC) connected in parallel via a coaxial switch and a Tee adaptor for an indication of 0.500000 V on the DMM and the emf e_x indicated by the nanovoltmeter is noted. The AC Calibrator is replaced by a calibrated DC calibrator and the DUC is disconnected. A DC voltage of positive polarity is applied to TVC and is adjusted so as to repeat a reading of e_x on the nanovoltmeter. The output of the DC calibration is noted as V_+. The polarity of the DC voltage is reversed and above process is repeated and DC calibrator output voltage V_- is recorded. The whole measurement process is repeated several times.

Mathematical model

The mathematical model used is

$$V_{AC} = (V_{DC} + \Delta V_{DC} + \Delta V_{th}) (1 + \delta) \qquad (C.62)$$

V_{AC} is the voltage estimated for an indicated value of 0.500000 V on DUC, $V_{DC} = v_+ + v_- / 2$ is average of two polarity DC voltage output of calibrator, δ is AC/DC transfer correction factor of the TVC at the frequency of calibration. ΔV_{DC} is error of DC calibrator due to its three months stability from the manufacturer's data, as the calibrator was calibrated three months before and ΔV_{th} is error due to thermal emf which comes from the fact that the polarity of DC voltage is reversed. This is very small as compared to 0.5 V being nearly equal to 1 μV. So, finally the equation becomes

$$V_{AC} = V_{DC} + \Delta V_{DC} + \delta V_{DC} + \delta \Delta V_{DC} \qquad (C.63)$$

The product $\delta \times \Delta V_{DC}$ is extremely small and is neglected. The assumption is that the drift in the values of δ is small and is also neglected and error of DMM in 1 volt range due to $\pm$ 1 count is $\pm$ 1μV and is also neglected. The precaution is that the interconnecting leads are coaxial shielded and are kept very small. The reference plane of measurement (mid point of Tee) is brought close to input plane of DMM (DUC). These precautions minimize the loading as well as transmission errors. At frequencies up to 10 kHz, with above precautions taken, the error contribution by above factors are very low ($\leq$ 2to 3 $\times 10^{-6}$) and can be neglected.

The inputs are :

1. The DC calibrator is regularly calibrated at intervals of six months. For the range of 1V the uncertainty in the calibrator from its calibration certificate is $\pm$ 5.8 $\times 10^{-6}$ at 95 % confidence level. Three months' stability data from the manufacturer's specifications is 5.0 $\times 10^{-6}$.

2. The AC /DC transfer correction factor for the thermal converter is + 0.008 %. The AC/DC transfer uncertainty is $\pm$ 0.01 % at 95 % confidence level.

The observations are average of two polarity DC voltages.

Table C.12: Experimental observations

Serial Number	Readings (V)
1	0.499986
2	0.499982
3	0.499991
4	0.499994
5	0.499993

Uncertainty evaluation

$$V_{AC} = V_{DC} + \Delta V_{DC} + \delta V_{DC} \quad \text{(C.64)}$$

For uncorrelated input quantities, the combined standard uncertainty is

$$u_c^2 = \sum_{i=1}^{n} \left[\frac{\delta f}{\delta x_i} \right]^2 u^2(x_i), \quad \text{(C.65)}$$

The components of total measurement uncertainty comprise of

u_1 (V) = DC calibrator's applied voltage uncertainty as mentioned in its calibration certificate

u_2 (V) = DC calibrator's uncertainty due to its stability

u_3 (V) = Uncertainty in the AC/DC transfer and

u_4 (V) = Uncertainty due to repeatability and the corresponding sensitivity coefficients are

$$c_1 = \frac{\delta V_{AC}}{\delta V_{DC}} = 1, c_2 = \frac{\delta V_{AC}}{\delta \Delta V_{DC}} = 1, and\ c_1 = \frac{\delta V_{AC}}{\delta \delta} = 1 \quad \text{(C.66)}$$

Type A evaluation

Mean DC Voltage = 0.499989 V,
Standard deviation = 0.0000005 V,
Standard deviation of mean or standard uncertainty

$$s(\bar{q}) = \frac{0.000005}{\sqrt{5}} = 2.23 \times 10^{-6} V \quad \text{(C.67)}$$

Degrees of freedom

$$\nu_i = 5 - 1 = 4 \quad \text{(C.68)}$$

Type B evaluation

1. Uncertainty of DC calibrator from its calibration certificate. The distribution is normal and the coverage factor for 95 % confidence level is 1.96.

$$u_1(V) = \frac{a_1}{1.96} = \frac{5.8}{1.96} = 2.96 \times 0.5 \ \mu V = 1.48 \ \mu V \quad \text{(C.69)}$$

Degrees of freedom = ∞

2. From DC calibrator's specifications, uncertainty due to 3 months stability data $a_2 = \pm 5.0 \times 10^{-6}$. For rectangular distribution, the standard uncertainty

$$u_2(V) = \frac{a_2}{\sqrt{3}} = \frac{5.0}{\sqrt{3}} = 2.89 \times 0.5 \ \mu V = 1.44 \ \mu V \quad \text{(C.70)}$$

Degrees of freedom = ∞

3. From AC/DC transfer at 95 % confidence level $a_3 = 100 \times 10^{-6}$, distribution is normal and coverage factor = 1.96

Standard uncertainty

$$u_3(V) = \frac{a_3}{1.96} = \frac{1 \times 100}{1.96} = 51.02 \times 0.5 \ \mu V = 25.5 \ \mu V \quad \text{(C.71)}$$

Degrees of freedom = ∞

Combined standard uncertainty

There is a dominant factor = 25.5 μV.

$$u_c = 25.5 + \sqrt{u_1^2 + u_2^2 + u_4^2} = 25.5 + \sqrt{(1.48)^2 + (1.44)^2 + (2.23)^2} \quad \text{(C.72)}$$

$$= \quad 25.5 + 3.04 = 28.5\ \mu\text{V} \quad \text{(C.73)}$$

Effective degrees of freedom (v_{eff})

$$v_{eff} = \frac{[u_c]^4}{\frac{(u_1)^4}{\infty} + \frac{(u_2)^4}{\infty} + \frac{(u_3)^4}{\infty} + \frac{(u_4)^4}{4}} \quad \text{(C.74)}$$

$$v_{eff} = \frac{[28.5]^4}{\frac{(1.48)^4}{\infty} + \frac{(1.44)^4}{\infty} + \frac{(2.23)^4}{\infty} + \frac{(25.5)^4}{4}} = \infty \quad \text{(C.75)}$$

Expanded uncertainty

For 95.45 % level of confidence, the coverage factor k = 2, thus

$$U = ku_c\ (V) = 2 \times 28.5 = 57\mu\text{V} \quad \text{(C.76)}$$

Table C.13: Uncertainty Budget:

Source of Uncertainty X_i	**Estimates x_i V**	**Limits $\pm \Delta x_i$ µV**	**Probability Distribution - Type A or B - Factor**	**Standard Uncertainty $u(x_i)$ µV**	**Sensitivity coefficient c_i**	**Uncertainty Contribution $u_i(y)$ µV**	**Degree of freedom v_i**
u_1	0.5	2.9	Normal -Type B -1.96	1.48	1.0	1.48	∞
u_2	0.0	2.5	Rectangular -Type B -√3	1.44	1.0	1.44	∞
u_3	0.0	50.0	Normal -Type B -1.96	25.5	1.0	25.5	∞
Repeatability			Normal -Type A			28.5	4
$u_c(V_{ac})$						28.5	∞
Expanded uncertainty			k=2.0			57.0	∞

Reporting of result

The measured average AC Voltage corresponding to 0.500000 V indicated by the DMM,

$$V_{AC} = 0.499989\ (1 + 0.000008)\ V \pm 57\ \mu V = 0.499993 V \pm 57\ \mu V \qquad (C.77)$$

C.7 Calibration of a RF Power Sensor at a Frequency of 18 GHz

Introduction

The measurement involves the calibration of an unknown power sensor against a standard power sensor as reference standard by substitution on a stable, monitored source of known reflection coefficient. The measurement is made in terms of calibration factor which for a matched source is defined as the ratio of incident power at the calibration frequency to the incident power at the reference frequency of 50 MHz under the condition that both incident powers give equal power sensor response.

The calibration factor K_x of the unknown power sensor is determined by

$$K_x = (K_s + D_s) \times \delta_{DC} \times \delta_M \times \delta_{REF,} \quad (C.78)$$

Where K_s = calibration factor of the standard sensor,
D_s = drift in standard sensor since the last calibration,
δ_{DC} = ratio of DC voltage outputs, and
δ_M = ratio of mismatch losses,
δ_{REF} = ratio of reference power source (short-term stability of 50 MHz).

Method of measurement

Five separate measurements were taken which involved disconnection and reconnection of both the unknown sensor and the standard sensor on a power transfer system. All measurements are made in terms of voltage ratios that are proportional to calibration factor.

None of uncertainty contributions is considered to be correlated.

Mismatch uncertainty:

As the source is not perfectly matched and the phase relation of the reflection coefficients of the source, the unknown and the standard sensors are not known, there is an uncertainty due to mismatch for each sensor at the calibration frequency as well as reference frequency. The corresponding limits of deviation is calculated from the well known formula:

Mismatch uncertainty = $\pm 2\Gamma_G\Gamma_S$ for the standard sensor. (C.79)

Mismatch uncertainty = $\pm 2\Gamma_G\Gamma_x$ for the unknown. (C.80)

Γ_G, Γ_s and Γ_x are the reflection coefficients for source, the standard and the unknown, respectively.

For this case

$$\Gamma_G \quad \text{at 50 MHz} = 0.02; \text{ at 18 GHz} = 0.06, \tag{C.81}$$

$$\Gamma_s \quad \text{at 50 MHz} = 0.02 \text{ ; at 18 GHz} = 0.09, \text{ and} \qquad \text{C.82)}$$

$$\Gamma_x \text{ at 50 MHz} = 0.02 \text{ ; at 18 GHz} = 0.10 \tag{C.83}$$

The long-term stability from the results of five annual calibrations was found to have limits not greater than ± 0.4 % per year.

The values of reflection coefficients are themselves uncertain. This uncertainty has been accounted for by adding it in quadrature with the actual measured values.

The instrumentation linearity uncertainty has been estimated to lie within ± 0.1 % from measurements against a reference attenuation standard up to ratios of 2:1 at confidence level of 95 %.

Reference source:

The ratio of power outputs of the reference source has been estimated to be 1.000 with deviations ± 0.004.

Standard sensor:

The standard sensor was calibrated 6 months ago. The value of calibration factor from its calibration certificate is

$$0.965 \pm 0.012$$

at confidence level of 95 %.

Uncertainty evaluation

Type A evaluation

The measured values of calibration factor for the unknown power sensor are shown in Table (C.14).

The mean value is

$$\overline{K_x} = 0.9496 \cong 0.950, \tag{C.84}$$

Table C.14: Calibration factor for the unknown sensor

Number	Calibration Factor
1	0.958
2	0.946
3	0.951
4	0.950
5	0.943

The experimental standard deviation [s(K_x)],

$$s(K_x) = 0.0057 \quad \text{(C.85)}$$

Standard uncertainty in $u(s)K_x$ is

$$u(K_x) = \frac{0.0057}{\sqrt{5}} = 0.0025 \quad \text{(C.86)}$$

Degree of freedom = 5 - 1 = 4.

Type B evaluation

1. Uncertainty reported in calibration certificate of the standard sensor = ± 0.012 at the confidence level of 95 %. The standard uncertainty u(K_s) is

   u(K_s) = 0.012 / 1.96 = 0.0061, (C.87)

   Degrees of freedom = ∞

2. Uncertainty in drift in calibration factor since its last calibration is ± 0.002. This is a rectangular distribution and the standard uncertainty,

   $$u(D_s) = \frac{0.002}{\sqrt{3}} = 0.0012 \text{ , and} \quad \text{(C.88)}$$

   Degrees of freedom = ∞

3. Uncertainty due to the stability of 50 MHz reference source is ± 0.004. This is a rectangular distribution and the standard uncertainty u (δ_{REF})

$$u(\delta_{REF}) = \frac{0.004}{\sqrt{3}} = 0.0023, and \quad \text{(C.89)}$$

Degrees of freedom = ∞

4. Uncertainty due to the instrument linearity is ± 0.001. This is a normal distribution and the standard Uncertainty u (δ_{DC})

$$u(\delta_{DC}) = \frac{0.001}{1.96} = 0.0023, and \tag{C.90}$$

Degrees of freedom = ∞

5. Uncertainty due to mismatch:

a) Standard sensor at 50 MHz = ± 0.0008
b) unknown sensor at 50 MHz = ± 0.0008
c) Standard sensor at 18 GHz = ± 0.0108
d) unknown sensor at 18 GHz = ± 0.012

This is U-shaped and the corresponding associated standard uncertainty figures are:

$$u(M_s) = \frac{0.0008}{\sqrt{2}} = 0.00056 \; at \; 50MHz \tag{C.91}$$

$$u(M_x) = \frac{0.0008}{\sqrt{2}} = 0.00056 \; at \; 50MHz \tag{C.92}$$

$$u(M_s) = \frac{0.0108}{\sqrt{2}} = 0.0076 \; at \; 18GHz \tag{C.93}$$

$$u(M_x) = \frac{0.012}{\sqrt{2}} = 0.0085 \; at \; 18GHz \tag{C.94}$$

Combined standard uncertainty:

$$u^2_c (K_x) = u^2 (K_s) + u^2 (D_s) + u^2 (\delta_{REF}) + u^2 (\delta) + u^2 (M_{SX}) + u^2 (K_r) \tag{C.95}$$

$$u^2_c (K_x) = (0.0061)^2 + (0.0012)^2 + (0.0023)^2 + (0.0005)^2 + 2(0.0056)^2 + (0.0076)^2 + (0.0085)^2 + (0.0025)^2$$

or,

$$u_c(K_x) \approx 0.0134 \tag{C.96}$$

Effective degrees of freedom ν_{eff}

From Welch-Satterthwaite formula, $\nu_{eff \, is}$ estimated and is found to be approximately 3301 or ∞.

Table C.15: Uncertainty Budget:

Source of Uncertainty X_i	Estimates x_i	Limits $\pm \Delta x_i$	Probability Distribution - Type A or B	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient c_i	Uncertainty Contribution $u_i(y)$	Degree of Freedom v_i
K_s	0.965	0.012	Normal -Type B	0.0061	1.0	0.0061	∞
D_s	0.002	0.002	Rectangular -Type B	0.0012	1.0	0.0012	∞
δ_{DC}	1.0	0.001	Normal -Type B	0.0005	1.0	0.0005	∞
δ_{REF}	1.0	0.004	Rectangular -Type B	0.0023	1.0	0.0023	∞
Mismatch at 50 MHz							
Γ_S	0.0	0.0008	-U shaped	0.00056	1.0	0.00056	∞
Γ_X	0.0	0.0008	-do-	0.00056	1.0	0.00056	∞
Mismatch at 18 GHz							
Γ_S	0.0	0.0108	-do-	0.0076	1.0	0.0076	∞
Γ_X	0.0	0.012	-do- -Type B	0.0085	1.0	0.0085	∞
Repeatability			Normal -Type A √ 5	0.0025	1.0	0.0025	4
$u_c(K_x)$						0.0134	3301
Expanded uncertainty			k = 1.96			0.0263	∞

Expanded uncertainty U (K_x)

$$U(K_x) = 0.0134 \times 2 = 0.0268 \approx 0.027 \quad (C.97)$$

Reported Result:

The calibration factor of unknown power sensor at 18 GHz is 0.450 ± 0.027

The reported expanded uncertainty of measurement is stated as the combined standard uncertainty multiplied by the coverage factor 1.96, which for a normal distribution corresponds to a coverage probability or confidence level of 95 %.

C.8 Calibration of 4 ½ digital multimeter for its 100 V range

Introduction

We discuss the method of calibration of a 4 ½ digital multimeter for its 100 V range with the application of 10 V from calibrated direct volt calibrator.

Mathematical model

The mathematical model used for the evaluation:

$$V_x = V_s + \Delta V_x \qquad \text{(C.98)}$$

Where

V_x = Voltage indicated in the DMM

V_s = Voltage applied from the calibrator

V_s = Error of the multimeter

Some simplifying assumptions have been made:

(i) errors due to loading and wire leads or connections are considered negligible, and

(ii) all input quantities are uncorrelated.

We have the following inputs:

(A) The calibrator is calibrated regularly at the interval of six months. For the range of 10 V, the specifications are: Resolution = 10 μV with the uncertainty at 99 % level of confidence as (4.5 $\times$ 10^{-6} of output + 100 μV),

$$V_x = V_s + \Delta V_x \qquad \text{(C.99)}$$

(B) 4½ digital multimeter specification are: for range 100 V full display is 99.99 with resolution = 10 mV and the uncertainty at 99 % level of confidence as $\pm$ (10^{-5} of reading + 0.2 $\times$ 10^{-5} of full scale).

Observation:

Applied voltage from calibrator	Indicated voltage in DMM
10.00000 V	10.01 V

Even after repetition of the readings the multimeter reading indicating the same value or $\pm$ 1 due to digitizing process. This is due to better accuracy of the reference standard (calibrator).

Combined standard uncertainty

For uncorrelated input quantities, the combined standard uncertainty is

$$u_c^2 = \sum_{i=1}^{N} \left[\left(\frac{\delta f}{\delta x_i} \right) \right]^2 u^2 (x_i) \qquad \text{(C.100)}$$

The components of the total measurement uncertainty consist of

u_1 (V) = the calibrator's applied voltage uncertainty
u_2 (V) = multimeter's random effect uncertainty

Corresponding sensitivity coefficients are:

$$c_1 = \frac{\partial V_x}{\partial V_s} = 1 \quad \textbf{and} \quad c_2 = \frac{\partial V_x}{\partial \Delta V_x} = 1 \qquad \text{(C.101)}$$

Evaluation of uncertainty components:

Type A evaluation:

Even after repetition of the readings the multimeter reading indicating the same value or ±1 due to digitizing process. This is due to better accuracy of the reference standard (calibrator). In this case, type A uncertainty can be assumed as negligible and the repeatability uncertainty can be treated as type B uncertainty using the resolution error of the multimeter.

Type B evaluation:

(I) The uncertainty in applied voltage from the calibrator is

$a_1 = 4.5 \times 10^{-6}$ OF OUTPUT + 100 μV

$= (4.5 \times 10 + 100)$ μV $= 145$ μV (C. 102)

At 99 % confidence level assuming normal distribution, coverage factor k = 2.58, the standard uncertainty in applied voltage is

u_1 (V) = a_1/2.58 = (145 / 2.58) μV = 56.20 μV (C.103)

Degree of freedom is = $v_1 = \infty$ (C.104)

(II) From multimeter specification in the 100 V range, resolution is 10 mV (i.e. 1 count). Since the reading was unchanged and assuming the limit to be half the counts.

$$a_2 = \frac{10}{2} mV = 5mV \qquad \text{(C.105)}$$

For rectangular distribution, the standard uncertainty due to the resolution uncertainty of the multimeter is:

$$u_2 V = \frac{a_2}{\sqrt{3}} = \frac{5}{\sqrt{3}} mV = 2886.75 \quad V \qquad \text{(C.106)}$$

Degree of freedom is = $v_2 = \infty$ (C.107)

Combined standard uncertainty

$$u_c(V) = \sqrt{u_1^2(V) + u_2^2(V)} = 2.89mV \qquad \text{(C.108)}$$

Effective degree of freedom $v_{eff} = \infty$ as $v_1 = \infty$ and $v_2 = \infty$

Expanded uncertainty (U)

For 95.45 % level of confidence the coverage factor, k = 2, thus

U = ku_c (V) = 2 x 2.89 mV = 5.78 mV

Result:

The measured average voltage of the unknown cell is 10.01 V ± 5.78 mV.

The reported expanded uncertainty in measurement is stated as the standard uncertainty in measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95.45 %

Table C. 16: Uncertainty Budget:

Source of Uncertainty X_i	**Estimates x_i**	**Limits $\pm \Delta x_i$**	**Probability Distribution - type A or B**	**Standard Uncertainty $u(x_i)$**	**Sensitivity coefficient c_i**	**Uncertainty contribution $u_i(y)$**	**Degree of Freedom v_i**
V_s	10.00 V		Normal – Type B - 2.5	56.2 μV	1.0	56.2 μV	∞
ΔV_x	0.01 V	5 mV	Rectangular - Type B - $\sqrt{3}$	2886.25 μV	1.0	2886.75 μV	∞
Repeatability			Normal - Type A	0	0	0	
$u_c(V_x)$						2.89 mV	∞
Expanded Uncertainty			k = 2			5.78 mV	∞

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